Chapter 6

6.1 a Relative frequency approach
b If the conditions today repeat themselves an infinite number of days rain will fall on 10% of the next days.

6.2 a Subjective approach
b If all the teams in major league baseball have exactly the same players the New York Yankees will win 25% of all World Series.

6.3 a \{a is correct, b is correct, c is correct, d is correct, e is correct\}
b \(P(a \text{ is correct}) = P(b \text{ is correct}) = P(c \text{ is correct}) = P(d \text{ is correct}) = P(e \text{ is correct}) = 0.2\)
c Classical approach
d In the long run all answers are equally likely to be correct.

6.4 a Subjective approach
b The Dow Jones Industrial Index will increase on 60% of the days if economic conditions remain unchanged.

6.5 a \(P(\text{even number}) = P(2) + P(4) + P(6) = 1/6 + 1/6 + 1/6 = 3/6 = 1/2\)
b \(P(\text{number less than or equal to 4}) = P(1) + P(2) + P(3) + P(4) = 1/6 + 1/6 + 1/6 + 1/6 = 4/6 = 2/3\)
c \(P(\text{number greater than or equal to 5}) = P(5) + P(6) = 1/6 + 1/6 = 2/6 = 1/3\)

6.6 \{Adams wins. Brown wins, Collins wins, Dalton wins\}

6.7a \(P(\text{Adams loses}) = P(\text{Brown wins}) + P(\text{Collins wins}) + P(\text{Dalton wins}) = 0.09 + 0.27 + 0.22 = 0.58\)
b \(P(\text{either Brown or Dalton wins}) = P(\text{Brown wins}) + P(\text{Dalton wins}) = 0.09 + 0.22 = 0.31\)
c \(P(\text{either Adams, Brown, or Collins wins}) = P(\text{Adams wins}) + P(\text{Brown wins}) + P(\text{Collins wins}) = 0.42 + 0.09 + 0.27 = 0.78\)

6.8 a \{0, 1, 2, 3, 4, 5\}
b \{4, 5\}
c \(P(5) = 0.10\)
d \(P(2, 3, \text{ or } 4) = P(2) + P(3) + P(4) = 0.26 + 0.21 + 0.18 = 0.65\)
e \(P(6) = 0\)

6.9 \{Contractor 1 wins, Contractor 2 wins, Contractor 3 wins\}

6.10 \(P(\text{Contractor 1 wins}) = 2/6, P(\text{Contractor 2 wins}) = 3/6, P(\text{Contractor 3 wins}) = 1/6\)
6.11 a \{\text{Shopper pays cash, shopper pays by credit card, shopper pays by debit card}\}
b \ \ \ \ P(\text{Shopper pays cash}) = .30, \ P(\text{Shopper pays by credit card}) = .60, \ P(\text{Shopper pays by debit card}) = .10
c Relative frequency approach

6.12 a \ P(\text{shopper does not use credit card}) = P(\text{shopper pays cash}) + P(\text{shopper pays by debit card})
\quad = .30 + .10 = .40
b \ P(\text{shopper pays cash or uses a credit card}) = P(\text{shopper pays cash}) + P(\text{shopper pays by credit card})
\quad = .30 + .60 = .90

6.13 \{\text{single, divorced, widowed}\}

6.14 a \ P(\text{single}) = .15, \ P(\text{married}) = .50, \ P(\text{divorced}) = .25, \ P(\text{widowed}) = .10
b Relative frequency approach

6.15 a \ P(\text{single}) = .15
b \ P(\text{adult is not divorced}) = P(\text{single}) + P(\text{married}) + P(\text{widowed}) = .15 + .50 + .10 = .75
c \ P(\text{adult is either widowed or divorced}) = P(\text{divorced}) + P(\text{widowed}) = .25 + .10 = .35

6.16 \ P(A_1 ) = .4 + .2 = .6, \ P(A_2 ) = .3 + .1 = .4. \ P(B_1 ) = .4 + .3 = .7, \ P(B_2 ) = .2 + .1 = .3.

6.17 \ P(A_1 ) = .1 + .2 = .3, \ P(A_2 ) = .3 + .1 = .4, \ P(A_3 ) = .2 + .1 = .3.
P(\ B_1 ) = .1 + .3 + .2 = .6, \ P(\ B_2 ) = .2 + .1 + .1 = .4.

6.18 a \ P(A_1 \mid B_1 ) = \frac{P(A_1 \text{ and } B_1 )}{P(B_1 )} = \frac{.4}{.7} = .57
b \ P(A_2 \mid B_1 ) = \frac{P(A_2 \text{ and } B_1 )}{P(B_1 )} = \frac{.3}{.7} = .43
c Yes. It is not a coincidence. Given \ B_1 \ the events \ A_1 \ and \ A_2 \ constitute the entire sample space.

6.19 a \ P(A_1 \mid B_2 ) = \frac{P(A_1 \text{ and } B_2 )}{P(B_2 )} = \frac{.2}{.3} = .67
b \ P(B_2 \mid A_1 ) = \frac{P(A_1 \text{ and } B_2 )}{P(A_1 )} = \frac{.2}{.6} = .33
c One of the conditional probabilities would be greater than 1, which is not possible.

6.20 The events are not independent because \ P(A_1 \mid B_2 ) \neq \ P(A_1 ) .
6.21 a \( P(A_1 \text{ or } B_1) = P(A_1) + P(B_1) - P(A_1 \text{ and } B_1) = .6 + .7 - .4 = .9 \)

b \( P(A_1 \text{ or } B_2) = P(A_1) + P(B_2) - P(A_1 \text{ and } B_2) = .6 + .3 - .2 = .7 \)

c \( P(A_1 \text{ or } A_2) = P(A_1) + P(A_2) = .6 + .4 = 1 \)

6.22 \( P(A_1 \mid B_1) = \frac{P(A_1 \text{ and } B_1)}{P(B_1)} = \frac{.20}{.20 + .15} = .571; P(A_1) = .20 + .60 = .80 \); the events are dependent.

6.23 \( P(A_1 \mid B_1) = \frac{P(A_1 \text{ and } B_1)}{P(B_1)} = \frac{.20}{.20 + .60} = .25; P(A_1) = .20 + .05 = .25 \); the events are independent.

6.24 \( P(A_1) = .20 + .25 = .45, P(A_2) = .15 + .25 = .40, P(A_3) = .10 + .05 = .15. \)

\( P(B_1) = .20 + 15 + .10 = .45, P(B_2) = .25 + .25 + .05 = .55. \)

6.25 a \( P(A_2 \mid B_2) = \frac{P(A_2 \text{ and } B_2)}{P(B_2)} = \frac{.25}{.55} = .455 \)

b \( P(B_2 \mid A_2) = \frac{P(A_2 \text{ and } B_2)}{P(A_2)} = \frac{.25}{.40} = .625 \)

c \( P(B_1 \mid A_2) = \frac{P(A_2 \text{ and } B_1)}{P(A_2)} = \frac{.15}{.40} = .375 \)

6.26 a \( P(A_1 \text{ or } A_2) = P(A_1) + P(A_2) = .45 + .40 = .85 \)

b \( P(A_2 \text{ or } B_2) = P(A_2) + P(B_2) - P(A_2 \text{ and } B_2) = .40 + .55 - .25 = .70 \)

c \( P(A_3 \text{ or } B_1) = P(A_3) + P(B_1) - P(A_3 \text{ and } B_1) = .15 + .45 - .10 = .50 \)

6.27 a \( P(\text{debit card}) = .04 + .18 + .14 = .36 \)

b \( P(\text{over }$100 \mid \text{credit card}) = \frac{P(\text{credit card and over }$100)}{P(\text{credit card})} = \frac{.23}{.03 + .21 + .23} = .49 \)

c \( P(\text{credit card or debit card}) = P(\text{credit card}) + P(\text{debit card}) = .47 + .36 = .83 \)

6.28 a \( P(\text{promoted} \mid \text{female}) = \frac{P(\text{promoted and female})}{P(\text{female})} = \frac{.05}{.05 + .12} = .294 \)

b \( P(\text{promoted} \mid \text{male}) = \frac{P(\text{promoted and male})}{P(\text{male})} = \frac{.15}{.15 + .68} = .181 \)

c Yes, because promotion and gender are not independent events.
6.29 a \( P(\text{voted}) = .25 + .18 = .43 \)

b \( P(\text{voted} | \text{female}) = \frac{P(\text{voted and female})}{P(\text{female})} = \frac{.25}{.25 + .33} = .31 \), \( P(\text{voted}) = .43 \), Subject to rounding the events are independent.

6.30 a \( P(\text{He is a smoker}) = .10 + .21 = .31 \)

b \( P(\text{He does not have lung disease}) = .21 + .66 = .87 \)

c \( P(\text{He has lung disease} | \text{he is a smoker}) = \frac{P(\text{he has lung disease and he is a smoker})}{P(\text{he is a smoker})} = \frac{.10}{.31} = .323 \)

d \( P(\text{He has lung disease} | \text{he does not smoke}) = \frac{P(\text{he has lung disease and he does not smoke})}{P(\text{he does not smoke})} = \frac{.03}{.69} = .044 \)

6.31 The events are dependent because \( P(\text{he has lung disease}) = .13 \), \( P(\text{he has lung disease} | \text{he is a smoker}) = .323 \)

6.32 a \( P(\text{manual} | \text{math-stats}) = \frac{P(\text{manual and math-stats})}{P(\text{math-stats})} = \frac{.23}{.23 + .36} = .390 \)

b \( P(\text{computer}) = .36 + .30 = .66 \)

c No, because \( P(\text{manual}) = .23 + .11 = .34 \), which is not equal to \( P(\text{manual} | \text{math-stats}) \).

6.33 a \( P(\text{customer will return and good rating}) = .35 \)

b \( P(\text{good rating} | \text{will return}) = \frac{P(\text{good rating and will return})}{P(\text{will return})} = \frac{.35}{.02 + .08 + .35 + .20} = \frac{.35}{.65} = .538 \)

c \( P(\text{will return} | \text{good rating}) = \frac{P(\text{good rating and will return})}{P(\text{good rating})} = \frac{.35}{.35 + .14} = \frac{.35}{.49} = .714 \)

d (a) is the joint probability and (b) and (c) are conditional probabilities

6.34 a \( P(\text{ask} | \text{male}) = \frac{P(\text{ask and male})}{P(\text{male})} = \frac{.12}{.23 + .12 + .15} = \frac{.12}{.50} = .24 \)

b \( P(\text{consult a map}) = .23 + .14 = .37 \)

c No, because \( P(\text{consult map} | \text{male}) = \frac{P(\text{consult a map and male})}{P(\text{male})} = \frac{.23}{.23 + .12 + .15} = \frac{.25}{.50} = .46 \), which is not equal to \( P(\text{consult map}) \)

6.35 a \( P(\text{ulcer}) = .03 + .03 + .03 + .04 = .13 \)
b $P(\text{ulcer} \mid \text{none}) = \frac{P(\text{ulcer and none})}{P(\text{none})} = \frac{.03}{.03 + .20} = \frac{.03}{.23} = .130$

c $P(\text{none} \mid \text{ulcer}) = \frac{P(\text{ulcer and none})}{P(\text{ulcer})} = \frac{.03}{.03 + .03 + .04} = \frac{.03}{.13} = .231$

d $P(\text{two} \mid \text{no ulcer}) = \frac{P(\text{no ulcer and two})}{P(\text{no ulcer})} = \frac{.32}{.20 + .19 + .32 + .16} = \frac{.32}{.87} = .368$

6.36 a $P(\text{remember}) = .12 + .18 = .30$

b $P(\text{remember} \mid \text{violent}) = \frac{P(\text{remember and violent})}{P(\text{violent})} = \frac{.12}{.12 + .38} = \frac{.12}{.50} = .24$

c Yes, the events are dependent.

6.37 a $P(\text{above average} \mid \text{murderer}) = \frac{P(\text{above average and murderer})}{P(\text{murderer})} = \frac{.27}{.27 + .21} = \frac{.27}{.48} = .563$

b No, because $P(\text{above average}) = .27 + .24 = .51$, which is not equal to $P(\text{above average testosterone} \mid \text{murderer})$.

6.38 a $P(\text{uses a spreadsheet}) = .311 + .312 = .623$

b $P(\text{uses a spreadsheet} \mid \text{male}) = \frac{P(\text{uses a spreadsheet and male})}{P(\text{male})} = \frac{.312}{.312 + .168} = \frac{.312}{.480} = .650$

c b $P(\text{female} \mid \text{uses a spreadsheet}) = \frac{P(\text{uses a spreadsheet and female})}{P(\text{uses a spreadsheet})} = \frac{.311}{.311 + .209} = \frac{.311}{.520} = .598$

6.39 No, because $P(\text{uses a spreadsheet}) \neq P(\text{uses a spreadsheet} \mid \text{male})$

6.40 a $P(\text{under 20}) = .2307 + .0993 + .5009 = .8309$

b $P(\text{retail}) = .5009 + .0876 + .0113 = .5998$

c $P(20 \text{ to } 99 \mid \text{construction}) = \frac{P(20 \text{ to } 99 \text{ and construction})}{P(\text{construction})} = \frac{.0189}{.2307 + .0189 + .0019} = \frac{.0189}{.2515} = .0751$

6.41 a $P(\text{provided by employer}) = .166 + .195 + .230 = .591$

b $P(\text{provided by employer} \mid \text{professional/technical}) = \frac{P(\text{provided by employer and professional/technical})}{P(\text{professional/technical})} = \frac{.166}{.166 + .094} = \frac{.166}{.260} = .638$

c $P(\text{provided by employer and blue-collar/services}) = \frac{P(\text{provided by employer and blue-collar/services})}{P(\text{blue-collar/services})} = \frac{.230}{.230 + .180} = \frac{.230}{.410} = .561$
6.42 \ a \ P(\text{new} \ | \ \text{overdue}) = \frac{P(\text{new and overdue})}{P(\text{overdue})} = \frac{.08}{.08 + .50} = .138

\ b \ P(\text{overdue} \ | \ \text{new}) = \frac{P(\text{new and overdue})}{P(\text{new})} = \frac{.08}{.08 + .13} = .381

\ c \ Yes, \ because \ P(\text{new}) = .21 \neq P(\text{new} \ | \ \text{overdue})

6.43 \ P(\text{purchase} \ | \ \text{see ad}) = \frac{P(\text{purchase and see ad})}{P(\text{see ad})} = \frac{.18}{.18 + .42} = \frac{.18}{.60} = .30; \ P(\text{purchase} \ | \ \text{do not see ad}) = \frac{P(\text{purchase and do not see ad})}{P(\text{do not see ad})} = \frac{.12}{.12 + .28} = \frac{.12}{.40} = .30; \ the \ ads \ are \ not \ effective

6.44 \ a \ P(\text{unemployed} \ | \ \text{high school graduate}) =
\frac{P(\text{unemployed and high school graduate})}{P(\text{high school graduate})} = \frac{.0128}{.3108 + .0128} = \frac{.0128}{.3236} = .0396

\ b \ P(\text{employed}) = .0975 + .3108 + .1785 + .0849 + .1959 + .0975 = .9651

\ c \ P(\text{advanced degree} \ | \ \text{unemployed}) =
\frac{P(\text{advanced degree and unemployed})}{P(\text{unemployed})} = \frac{.0015}{.0080 + .0128 + .0062 + .0023 + .0041 + .0015} = \frac{.0015}{.0349} = .0430

\ d \ P(\text{not a high school graduate}) = .0975 + .0080 = .1055

6.45 \ a \ P(\text{fully repaid}) = .17 + .66 = .83

\ b \ P(\text{fully repaid} \ | \ \text{under 400}) = \frac{P(\text{fully repaid and under 400})}{P(\text{under 400})} = \frac{.17}{.17 + .30} = \frac{.17}{.47} = .367

\ c \ P(\text{fully repaid} \ | \ \text{400 or more}) = \frac{P(\text{fully repaid and 400 or more})}{P(\text{400 or more})} = \frac{.66}{.66 + .44} = \frac{.66}{.70} = .943

\ d \ No, \ because \ P(\text{fully repaid}) \neq P(\text{fully repaid} \ | \ \text{under 400})

6.46 \ a \ P(\text{bachelor's degree} \ | \ \text{west}) =
\frac{P(\text{bachelor's degree and west})}{P(\text{west})} = \frac{.0418}{.0359 + .0608 + .0456 + .0181 + .0418 + .0180} = \frac{.0418}{.2202} = .1973

\ b \ P(\text{northwest} \ | \ \text{high school graduate}) =
\frac{P(\text{northwest and high school graduate})}{P(\text{high school graduate})} = \frac{.0711}{.0711 + .0843 + .1174 + .0608} = \frac{.0711}{.3336} = .2131

\ c \ P(\text{south}) = .0683 + .1174 + .0605 + .0248 + .0559 + .0269 = .3538
6.50

Joint events | Probabilities
--- | ---
A and B | 
B|A 0.3 | (0.6)(0.3) = 0.18
A and B^c | 
B|A^c 0.7 | (0.6)(0.7) = 0.42
A^c and B | 
B|A^c 0.3 | (0.4)(0.3) = 0.12
A^c and B^c | 
B|A^c 0.7 | (0.4)(0.7) = 0.28

6.51

Joint events | Probabilities
--- | ---
A and B | 
B|A 0.3 | (0.4)(0.3) = 0.12

6.52

Joint events | Probabilities
--- | ---
R and R | 
R 0.9 | (0.9)(0.9) = 0.81
L 0.1 | 
R 0.9 | 
L 0.1 | 
L 0.1 | 
L 0.1 | 
R 0.9 | 
L 0.1 | 
L 0.1 | 

a P(R and R) = .81
b P(L and L) = .01
c \[ P(R \text{ and } L) + P(L \text{ and } R) = .09 + .09 = .18 \]

d \[ P(R \text{ and } L) + P(L \text{ and } R) + P(R \text{ and } R) = .09 + .09 + .81 = .99 \]

6.53 a & b

<table>
<thead>
<tr>
<th>Joint events</th>
<th>Probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>RRR (0.9)(0.9)(0.9) = 0.729</td>
<td></td>
</tr>
<tr>
<td>RRL (0.9)(0.9)(0.1) = 0.081</td>
<td></td>
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<tr>
<td>RLR (0.9)(0.1)(0.9) = 0.081</td>
<td></td>
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<tr>
<td>RLL (0.9)(0.1)(0.1) = 0.009</td>
<td></td>
</tr>
<tr>
<td>LRR (0.1)(0.9)(0.9) = 0.081</td>
<td></td>
</tr>
<tr>
<td>LRL (0.1)(0.9)(0.1) = 0.009</td>
<td></td>
</tr>
<tr>
<td>LLR (0.1)(0.1)(0.9) = 0.009</td>
<td></td>
</tr>
<tr>
<td>LLL (0.1)(0.1)(0.1) = 0.001</td>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>c</th>
<th>0 right-handers</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 right-hander</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>2 right-handers</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>3 right-handers</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

\[ P(0 \text{ right-handers}) = .001 \]

\[ P(1 \text{ right-hander}) = 3(.009) = .027 \]

\[ P(2 \text{ right-handers}) = 3(.081) = .243 \]

\[ P(3 \text{ right-handers}) = .729 \]
6.54a

<table>
<thead>
<tr>
<th>Joint events</th>
<th>Probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>RR 8/99</td>
<td>RR (90/100)(89/99) = 0.8091</td>
</tr>
<tr>
<td>LR 10/99</td>
<td>RL (90/100)(10/99) = 0.0909</td>
</tr>
<tr>
<td>L 10/100</td>
<td>LL (10/100)(9/99) = 0.0909</td>
</tr>
</tbody>
</table>

b $P(\text{RR}) = 0.8091$
c $P(\text{LL}) = 0.0091$
d $P(\text{RL}) + P(\text{LR}) = 0.0909 + 0.0909 = 0.1818$
e $P(\text{RL}) + P(\text{LR}) + P(\text{RR}) = 0.0909 + 0.0909 + 0.8091 = 0.9909$

6.55a

<table>
<thead>
<tr>
<th>Joint events</th>
<th>Probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>RR 88/98</td>
<td>RRR 0.7265</td>
</tr>
<tr>
<td>LR 10/98</td>
<td>RRL 0.0826</td>
</tr>
<tr>
<td>RL 89/98</td>
<td>RLR 0.0826</td>
</tr>
<tr>
<td>L 89/99</td>
<td>RLL 0.0083</td>
</tr>
<tr>
<td>L 99/99</td>
<td>LRR 0.0826</td>
</tr>
<tr>
<td>L 99/98</td>
<td>LRL 0.0083</td>
</tr>
<tr>
<td>L 98/98</td>
<td>LLR 0.0083</td>
</tr>
<tr>
<td>L 98/98</td>
<td>LLL 0.0007</td>
</tr>
</tbody>
</table>

$P(0 \text{ right-handers}) = (10/100)(9/99)(8/98) = 0.0007$
$P(1 \text{ right-hander}) = 3(90/100)(10/99)(9/98) = 0.0249$
$P(2 \text{ right-handers}) = 3(90/100)(89/99)(10/98) = 0.2478$
$P(3 \text{ right-handers}) = (90/100)(89/99)(88/98) = 0.7265$
6.56

No Ans. 0.25
or Busy

Sale 0.10
Answer and Sale

Answer 0.75

No Sale 0.90

P(sale) = 0.075

6.57

<table>
<thead>
<tr>
<th>First contract</th>
<th>Second contract</th>
<th>Joint events</th>
<th>Probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>win 0.3</td>
<td>win</td>
<td>0.8</td>
<td>win and win</td>
</tr>
<tr>
<td></td>
<td>lose</td>
<td>win 0.2</td>
<td>win and lose</td>
</tr>
<tr>
<td></td>
<td>win</td>
<td>lose 0.5</td>
<td>lose and win</td>
</tr>
<tr>
<td></td>
<td>lose</td>
<td>lose 0.5</td>
<td>lose and lose</td>
</tr>
</tbody>
</table>

a P(win both) = .24
b P(lose both) = .35
c P(win only one) = .06 + .35 = .41
6.58

\[ P(D) = 0.02 + 0.045 = 0.065 \]

\[
\begin{align*}
F & \quad 0.10 \\
\quad & \\
F^c & \quad 0.90 \\
\quad & \\
D|F & \quad 0.20 \\
\quad & \\
D|F^c & \quad 0.05 \\
\quad & \\
\end{align*}
\]

\[ P(D) = 0.02 + 0.045 = 0.065 \]

6.59

\[
\begin{align*}
\text{Male} & \quad 0.42 \\
\quad & \\
\text{Female} & \quad 0.58 \\
\quad & \\
V|\text{Male} & \quad 0.43 \\
\quad & \\
V|\text{Female} & \quad 0.43 \\
\quad & \\
\text{Male and } V & \quad (0.42)(0.43) = 0.1806 \\
\quad & \\
\text{Female and } V & \quad (0.58)(0.43) = 0.2494 \\
\quad & \\
\end{align*}
\]

a \( P(\text{vote in last election and male}) = 0.1806 \)
b \( P(\text{vote in last election and female}) = 0.2494 \)
6.60

\[ P(\text{heart attack}) = 0.0594 + 0.0737 = 0.1331 \]

\[ P(\text{heart attack}) = 0.12 + 0.04 + 0.12 + 0.0075 + 0.04 + 0.0075 = 0.335 \]

6.61

\[ \text{Joint events} \quad \text{Probabilities} \]

\[ \begin{array}{ccc}
W & 0.8 & WW \\
B & 0.15 & WB \\
A & 0.05 & WA \\
W & 0.8 & BW \\
B & 0.15 & BB \\
A & 0.05 & BA \\
W & 0.8 & AW \\
B & 0.15 & AB \\
A & 0.05 & AA
\end{array} \]

\[ (0.80)(0.80) = 0.64 \quad (0.80)(0.15) = 0.12 \quad (0.80)(0.05) = 0.04 \quad (0.15)(0.80) = 0.12 \quad (0.15)(0.15) = 0.0225 \quad (0.15)(0.05) = 0.0075 \quad (0.05)(0.80) = 0.04 \quad (0.05)(0.15) = 0.0075 \quad (0.05)(0.05) = 0.0025 \]

Diversity index = \[ 0.12 + 0.04 + 0.0075 + 0.04 + 0.0075 = 0.335 \]
6.62

<table>
<thead>
<tr>
<th>Event</th>
<th>Joint Events</th>
<th>Probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pass</td>
<td>I and Pass</td>
<td>( (3000/7500)(0.57) = 0.228 )</td>
</tr>
<tr>
<td>Pass</td>
<td>II and Pass</td>
<td>( (2500/7500)(0.073) = 0.243 )</td>
</tr>
<tr>
<td>Pass</td>
<td>III and Pass</td>
<td>( (2000/7500)(0.85) = 0.227 )</td>
</tr>
</tbody>
</table>

\[ P(\text{pass}) = 0.228 + 0.243 + 0.227 = 0.698 \]

6.63

<table>
<thead>
<tr>
<th>Event</th>
<th>Joint Events</th>
<th>Probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>and G</td>
<td>( (0.76)(0.36) = 0.2736 )</td>
</tr>
<tr>
<td>F</td>
<td>and G</td>
<td>( (0.24)(0.32) = 0.0768 )</td>
</tr>
</tbody>
</table>

\[ P(\text{good}) = 0.2736 + 0.0764 = 0.3504 \]
6.64

\[
P(\text{myopic}) = P(\text{Some and Myopia}) + P(\text{No and Myopia}) = (0.28)(0.42) + (0.72)(0.21) = 0.1176 + 0.1512 = 0.2688
\]

6.65

\[
P(\text{does not have to be discarded}) = P(\text{F and R}) + P(\text{F and R}^c) = (0.22)(0.84) + (0.78)(0.16) = 0.1848 + 0.1248 = 0.9648
\]

6.66 Let \( A \) = mutual fund outperforms the market in the first year
\( B \) = mutual fund outperforms the market in the second year
\[P(\text{A and B}) = P(\text{A})P(\text{B | A}) = (0.15)(0.22) = 0.033\]

6.67 \( P(\text{wireless Web user uses it primarily for e-mail}) = 0.69\)
\( P(\text{3 wireless Web users use it primarily for e-mail}) = (0.69)(0.69)(0.69) = 0.3285\)

6.68 Define the events:
\( M \): The main control will fail.
\( B_1 \): The first backup will fail.
\( B_2 \): The second backup will fail.
The probability that the plane will crash is
\[
P(M \text{ and } B_1 \text{ and } B_2) = [P(M)][P(B_1)][P(B_2)]
\]
\[
= (.0001)(.01)(.01)
\]
\[
= .00000001
\]
We have assumed that the 3 systems will fail independently of one another.

6.69 Let A = DJIA increase and B = NASDAQ increase
\[
P(A) = .60 \text{ and } P(B | A) = .77
\]
\[
P(A \text{ and } B) = P(A)P(B | A) = (.60)(.77) = .462
\]

6.70

<table>
<thead>
<tr>
<th>Joint events</th>
<th>Probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>R and Increase</td>
<td>(0.30)(0.20) = 0.06</td>
</tr>
<tr>
<td>R and Increase</td>
<td>(0.70)(0.75) = 0.525</td>
</tr>
</tbody>
</table>

P(Increase) = .06 + .525 = .585

6.71 P(A and B) = .36, P(B) = .36 + .07 = .43
\[
P(A | B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{.36}{.43} = .837
\]

6.72 P(A and B) = .32, P(A^C and B) = .14, P(B) = .46, P(B^C) = .54

\[
a \ P(A | B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{.32}{.46} = .696
\]
\[
b \ P(A^C | B) = \frac{P(A^C \text{ and } B)}{P(B)} = \frac{.14}{.46} = .304
\]
\[
c \ P(A \text{ and } B^C) = .48; \ P(A | B^C) = \frac{P(A \text{ and } B^C)}{P(B^C)} = \frac{.48}{.54} = .889
\]
\[
d \ P(A^C \text{ and } B^C) = .06; \ P(A^C | B^C) = \frac{P(A^C \text{ and } B^C)}{P(B^C)} = \frac{.06}{.54} = .111
\]
6.73 Define events: A = crash with fatality, B = BAC is greater than .09
\[ P(A) = .01, \quad P(B | A) = .084, \quad P(B) = .12 \]
\[ P(A \text{ and } B) = (.01)(.084) = .00084 \]
\[ P(A | B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{.00084}{.12} = .007 \]

6.74 Define events: A = crash with fatality, B = BAC is greater than .09
\[ P(A) = .01, \quad P(B | A) = .084, \quad P(B) = .12 \]
\[ P(A \text{ and } B) = (.01)(.084) = .00084 \]
\[ P(A | B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{.00084}{.12} = .007 \]

6.74 P(F | D) = \frac{P(F \text{ and } D)}{P(D)} = \frac{.020}{.038} = .526

6.75 Define events: A = heart attack, B = periodontal disease
\[ P(A) = .10, \quad P(B | A) = .85, \quad P(B | A^c) = .29 \]
\[ P(A) = .10, \quad P(B | A) = .85, \quad P(B | A^c) = .29 \]
Joint events: \( P(B) = .085 + .261 = .346 \)
\[ P(A | B) = \frac{P(A \cap B)}{P(B)} = \frac{.085}{.346} = .246 \]
\[ P(A^c | B) = \frac{P(A^c \cap B)}{P(B)} = \frac{.261}{.346} = .754 \]

Define events: 
- \( A = \text{smoke} \)
- \( B_1 = \text{did not finish high school} \)
- \( B_2 = \text{high school graduate} \)
- \( B_3 = \text{some college, no degree} \)
- \( B_4 = \text{completed a degree} \)

**Exercise 6.45**: 
- \( P(B_1) = .1055 \)
- \( P(B_2) = .3236 \)
- \( P(B_3) = .1847 \)
- \( P(B_4) = .3862 \)

**Exercise 6.79**: 
- \( P(A | B_1) = .40 \)
- \( P(A | B_2) = .34 \)
- \( P(A | B_3) = .24 \)
- \( P(A | B_4) = .14 \)

From Exercise 6.45: 
- \( P(B_1) = .1055 \)
- \( P(B_2) = .3236 \)
- \( P(B_3) = .1847 \)
- \( P(B_4) = .3862 \)
6.80 Define events: A, B, C = airlines A, B, and C, D = on time

P(A) = .50, P(B) = .30, P(C) = .20, P(D | A) = .80, P(D | B) = .65, P(D | C) = .40

\[ P(D) = .40 + .195 + .08 = .675 \]

\[ P(A | D) = \frac{P(A \text{ and } D)}{P(D)} = \frac{.40}{.675} = .593 \]

6.81 Define events: A = win series, B = win first game
P(A) = .60, P(B | A) = .70, P(B | A^c) = .25

Joint events | Probabilities
---|---
A and B^c | (0.60)(0.30) = 0.18
A^c and B^c | (0.40)(0.75) = 0.30

P(B^c) = .18 + .30 = .48

P(A | B^c) = \frac{P(A \text{ and } B^c)}{P(B^c)} = \frac{.18}{.48} = .375

P(B^c) \times \ P(A) = 6.82

Joint events | Probabilities
---|---
PT and R | (0.35)(0.80) = 0.28
PT and R^c | (0.65)(0.08) = 0.052

P(P\text{T}) = .28 + .052 = .332

P(R | PT) = \frac{P(R \text{ and PT})}{P(PT)} = \frac{.28}{.332} = .843

6.83
$P(PT) = .0046 + .0269 = .0315$

$P(H | PT) = \frac{P(H \cap PT)}{P(PT)} = \frac{.0046}{.0315} = .1460$

6.84 Sensitivity = $P(PT | H) = .920$

Specificity = $P(NT | H^C) = .973$

Positive predictive value = $P(H | PT) = .1460$

Negative predictive value = $P(H^C | NT) = \frac{P(H^C \cap NT)}{P(NT)} = \frac{.9681}{.0046 + .9681} = \frac{.9681}{.9685} = .9996$

$P(PT) = .0164 + .6233 = .6397$

$P(H | PT) = \frac{P(H \cap PT)}{P(PT)} = \frac{.0164}{.6397} = .0255$
\[ P(NT) = .0036 + .3567 = .3603 \]
\[ P(C \mid PT) = \frac{P(C \text{ and } PT)}{P(PT)} = \frac{.0164}{.6397} = .0256 \]
\[ P(C \mid NT) = \frac{P(C \text{ and } NT)}{P(NT)} = \frac{.0036}{.3603} = .010 \]

6.86 a \( P(\text{Marketing-A}) = .06 + .23 = .29 \)

b \( P(\text{Marketing A} \mid \text{Statistics not A}) = \frac{P(\text{Marketing A and Statistics not A})}{P(\text{Statistics not A})} = \frac{.23}{.23 + .58} = \frac{.23}{.81} = .2840 \)

c No, the probabilities in (a) and (b) differ

6.87 Define events: \( A = \text{win contract A} \) and \( B = \text{win contract B} \)

<table>
<thead>
<tr>
<th>Joint events</th>
<th>Probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>( .3 )</td>
</tr>
<tr>
<td>( B \mid A )</td>
<td>( .4 )</td>
</tr>
<tr>
<td>( B \mid A^c )</td>
<td>( .6 )</td>
</tr>
<tr>
<td>( A^c )</td>
<td>( .7 )</td>
</tr>
<tr>
<td>( B \mid A^c )</td>
<td>( .2 )</td>
</tr>
<tr>
<td>( B \mid A^c )</td>
<td>( .8 )</td>
</tr>
</tbody>
</table>

a \( P(A \text{ and } B) = .12 \)

b \( P(A \text{ and } B^c) + P(A^c \text{ and } B) = .18 + .14 = .32 \)

c \( P(A \text{ and } B) + P(A \text{ and } B^c) + P(A^c \text{ and } B) = .12 + .18 + .14 = .44 \)

6.88 a \( P(\text{second}) = .05 + .14 = .19 \)

b \( P(\text{successful} \mid \geq -8 \text{ or less}) = \frac{P(\text{successful and } \geq -8 \text{ or less})}{P(\geq -8 \text{ or less})} = \frac{.15}{.15 + .14} = \frac{.15}{.29} = .517 \)

c No, because \( P(\text{successful}) = .66 + .15 = .81 \), which is not equal to \( P(\text{successful} \mid \geq -8 \text{ or less}) \).
6.89 Define events: A = woman, B = drug is effective

\[ P(B) = 0.528 + 0.221 = 0.749 \]

\[ P(A^c | B) = \frac{221}{749} = 0.295 \]

6.90 \[ P(A^c | B) = \frac{P(A^c \text{ and } B)}{P(B)} = \frac{221}{749} = 0.295 \]

6.91 P(Idle roughly)

\[ = P(\text{at least one spark plug malfunctions}) = 1 - P(\text{all function}) = 1 - (0.9^4) = 1 - 0.6561 = 0.3439 \]

6.92

\[ P(\text{no sale}) = 0.65 + 0.175 = 0.825 \]

6.93 a \[ P(pass) = 0.86 + 0.03 = 0.89 \]

b \[ P(pass \mid \text{miss 5 or more classes}) = \frac{P(pass \text{ and miss more classes})}{P(\text{miss 5 or more classes})} = \frac{0.03}{0.09 + 0.03} = \frac{0.03}{0.12} = 0.250 \]

c \[ P(pass \mid \text{miss less than 5 classes}) = \frac{P(pass \text{ and miss less than 5 classes})}{P(\text{miss less than 5 classes})} = \frac{0.86}{0.86 + 0.02} = \frac{0.86}{0.88} \approx 0.977 \]

d No since \( P(pass) \neq P(pass \mid \text{miss 5 or more classes}) \)
6.94

Joint events  Probabilities

\[ R \quad 0.27 \]
\[ D \quad 0.41 \]
\[ D^c \quad 0.59 \]
\[ R \quad 0.73 \]
\[ D^c \quad 0.69 \]

- a \( P(D) = P(R \text{ and } D) + P(R^c \text{ and } D) = 0.1107 + 0.2263 = 0.3370 \)
- \( P(R|D) = \frac{P(R \text{ and } D)}{P(D)} = \frac{0.1107}{0.3370} = 0.3285 \)
- b \( P(D^c) = P(R \text{ and } D^c) + P(R^c \text{ and } D^c) = 0.1593 + 0.5037 = 0.6630 \)
- \( P(R|D^c) = \frac{P(R \text{ and } D^c)}{P(D^c)} = \frac{0.1593}{0.6630} = 0.2403 \)

6.95

- a \( P(\text{excellent}) = 0.27 + 0.22 = 0.49 \)
- b \( P(\text{excellent} | \text{man}) = \frac{0.22}{0.22 + 0.10 + 0.12 + 0.06} = 0.44 \)
- c \( P(\text{man} | \text{excellent}) = \frac{P(\text{man and excellent})}{P(\text{excellent})} = \frac{0.22}{0.27 + 0.22} = \frac{0.22}{0.49} = 0.449 \)
- d No, since \( P(\text{excellent}) \neq P(\text{excellent} | \text{man}) \)
6.96

\[ P(R) = 0.0176 + 0.5888 = 0.6064 \]

\[ P(S \mid R) = \frac{2031.9097}{5888.0064} = 0.9710 \]

6.97 Define events: \( A_1 = \) Low-income earner, \( A_2 = \) medium-income earner, \( A_3 = \) high-income earner, \( B = \) die of a heart attack

\[
\begin{align*}
\text{Joint Events} & \quad \text{Probabilities} \\
R \mid S^c & \quad 0.22 \\
S^c & \quad 0.08 \\
& \quad R \mid S & \quad 0.64 \\
S & \quad 0.92 \\
\end{align*}
\]

\[ P(S \mid R) = \frac{P(S \text{ and } R)}{P(R)} = \frac{0.5888}{0.6064} = 0.9710 \]

\[ P(B^c) = 0.1848 + 0.4459 + 0.2790 = 0.9097 \]

\[ P(A_1 \mid B^c) = \frac{P(A_1 \text{ and } B^c)}{P(B^c)} = \frac{0.1848}{0.9097} = 0.2031 \]
6.98 Define the events: \(A_1\) = envelope containing two Maui brochures is selected, \(A_2\) = envelope containing two Oahu brochures is selected, \(A_3\) = envelope containing one Maui and one Oahu brochures is selected. \(B\) = a Maui brochure is removed from the selected envelope.

\[
P(B) = \frac{1}{3} + 0 + \frac{1}{6} = \frac{1}{2}
\]

\[
P(A \mid B) = \frac{A_1B}{P(B)} = \frac{1/3 \times 1}{1/2} = \frac{2}{3}
\]

6.99 Define events: \(A\) = purchase extended warranty, \(B\) = regular price

a \(P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{.21}{.21 + .57} = \frac{.21}{.78} = .2692\)

b \(P(A) = .21 + .14 = .35\)

c No, because \(P(A) \neq P(A \mid B)\)

6.100 Define events: \(A\) = company fail, \(B\) = predict bankruptcy
P(A) = .08, P(B | A) = .85, P(B^C | A^C) = .74

Joint events

<table>
<thead>
<tr>
<th>Event</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>A and B</td>
<td>(0.08)(0.85) = 0.068</td>
</tr>
<tr>
<td>A^C and B</td>
<td>(0.92)(0.26) = 0.2392</td>
</tr>
</tbody>
</table>

P(B) = .068 + .2392 = .3072

P(A | B) = P(A and B) / P(B) = .068 / .3072 = .2214

6.101 Define events: A = job security is an important issue, B = pension benefits is an important issue

P(A) = .74, P(B) = .65, P(A | B) = .60

a P(A and B) = P(B)P(A | B) = (.65)(.60) = .39

b P(A or B) = P(A) + P(B) - P(A and B) = .74 + .65 - .39 = 1

6.102 Probabilities of outcomes: P(HH) = .25, P(HT) = .25, P(TH) = .25, P(TT) = .25

P(TT | HH is not possible) = .25 / (0.25 + 0.25 + 0.25) = .333

6.103 P(T) = .5

Case 6.1

1. P(Curtain A) = 1/3, P(Curtain B) = 1/3

2. P(Curtain A) = 1/3, P(Curtain B) = 2/3

Case 6.2

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Probability of outcome</th>
<th>Bases Occupied</th>
<th>Outs</th>
<th>Probability of scoring</th>
<th>Joint Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.75</td>
<td>2nd</td>
<td>1</td>
<td>.42</td>
<td>.3150</td>
</tr>
<tr>
<td>2</td>
<td>.10</td>
<td>1st</td>
<td>1</td>
<td>.26</td>
<td>.0260</td>
</tr>
<tr>
<td>3</td>
<td>.10</td>
<td>none</td>
<td>2</td>
<td>.07</td>
<td>.0070</td>
</tr>
<tr>
<td>4</td>
<td>.05</td>
<td>1st and 2nd</td>
<td>0</td>
<td>.59</td>
<td>.0295</td>
</tr>
</tbody>
</table>

P(scoring) = .3775

Because the probability of scoring with a runner on first base with no outs (.39) is greater than the probability of scoring after bunting (.3775) you should not bunt.
Case 6.3

0 outs:
Probability of scoring any runs from first base = .39
Probability of scoring from second base = probability of successful steal × probability of scoring any runs from second base = (.68)(.57) = .3876
Decision: Do not attempt to steal.

1 out:
Probability of scoring any runs from first base = .26
Probability of scoring from second base = probability of successful steal × probability of scoring any runs from second base = (.68) × (.42) = .2856
Decision: Attempt to steal.

2 outs:
Probability of scoring any runs from first base = .13
Probability of scoring from second base = probability of successful steal × probability of scoring any runs from second base = (.68) × (.24) = .1632
Decision: Attempt to steal.

Case 6.4

Joint Events

\[
\begin{array}{c}
\text{D and PT} \\
\text{D and NT} \\
\text{D} \\
\text{D} \\
\text{NT} \\
\text{NT} \\
\text{D} \\
\text{D} \\
\text{NT} \\
\text{NT} \\
\end{array}
\]
Age 25: P(D) = 1/1,300
P(D and PT) = (1/1,300)(.624) = .00048
P(D and NT) = (1/1,300)(.376) = .00029
P(D^C and PT) = (1,299/1,300)(.04) = .03997
P(D^C and NT) = (1,299/1,300)(.96) = .95926
P(PT) = .00048 + .03997 = .04045
P(NT) = .00029 + .95926 = .95955
P(D | PT) = .00048/.04045 = .01187
P(D | NT) = .00029/.95955 = .00030

Age 30: P(D) = 1/900
P(D and PT) = (1/900)(.710) = .00079
P(D and NT) = (1/900)(.290) = .00032
P(D^C and PT) = (899/900)(.082) = .08190
P(D^C and NT) = (899/900)(.918) = .91698
P(PT) = .00079 + .08190 = .08269
P(NT) = .00032 + .91698 = .91730
P(D | PT) = .00079/.08269 = .00955
P(D | NT) = .00032/.91730 = .00035

Age 35: P(D) = 1/350
P(D and PT) = (1/350)(.731) = .00209
P(D and NT) = (1/350)(.269) = .00077
P(D^C and PT) = (349/350)(.178) = .17749
P(D^C and NT) = (349/350)(.822) = .81965
P(PT) = .00209 + .17749 = .17958
P(NT) = .00077 + .81965 = .82042
P(D | PT) = .00209/.17958 = .01163
P(D | NT) = .00077/.82042 = .00094

Age 40: P(D) = 1/100
P(D and PT) = (1/100)(.971) = .00971
P(D and NT) = (1/100)(.029) = .00029
P(D^C and PT) = (99/100)(.343) = .33957
P(D^C and NT) = (99/100)(.657) = .65043
P(PT) = .00971 + .33957 = .34928
\[ P(NT) = .00029 + .65043 = .65072 \]
\[ P(D \mid PT) = .00971/.34928 = .02780 \]
\[ P(D \mid NT) = .00029/.65072 = .00045 \]

**Age 45:**
\[ P(D) = 1/25 \]
\[ P(D \text{ and } PT) = (1/25)(.971) = .03884 \]
\[ P(D \text{ and } NT) = (1/25)(.029) = .00116 \]
\[ P(D^C \text{ and } PT) = (24/25)(.343) = .32928 \]
\[ P(D^C \text{ and } NT) = (24/25)(.657) = .63072 \]
\[ P(PT) = .03884 + .32928 = .36812 \]
\[ P(NT) = .00116 + .63072 = .63188 \]
\[ P(D \mid PT) = .03884/.36812 = .10551 \]
\[ P(D \mid NT) = .00116/.63188 = .00184 \]

**Age 49:**
\[ P(D) = 1/12 \]
\[ P(D \text{ and } PT) = (1/12)(.971) = .08092 \]
\[ P(D \text{ and } NT) = (1/12)(.029) = .00242 \]
\[ P(D^C \text{ and } PT) = (11/12)(.343) = .31442 \]
\[ P(D^C \text{ and } NT) = (11/12)(.657) = .60255 \]
\[ P(PT) = .08092 + .31442 = .39533 \]
\[ P(NT) = .00242 + .60255 = .60467 \]
\[ P(D \mid PT) = .08092/.39533 = .20468 \]
\[ P(D \mid NT) = .00242/.60467 = .00400 \]