9 Permanent-Magnet Synchronous and Brushless DC Motor Drives

- Two developments contributed to PM synchronous and brushless DC machines:

1. Armature placed on stator;
2. Excitation field on rotor with the PM poles

9.2 Permanent magnets and characteristics

- Permanent magnets

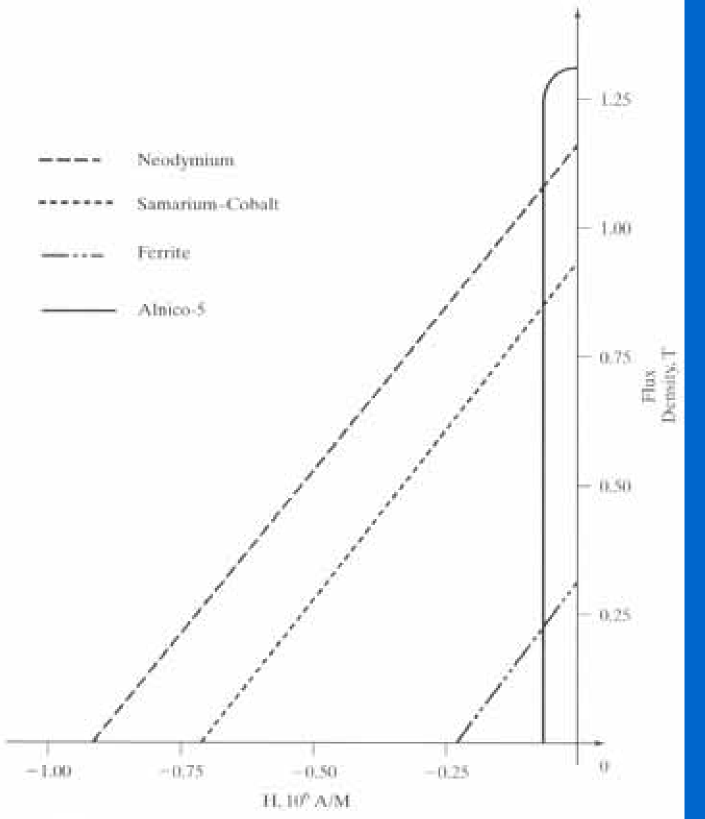


Figure 9.1 Second-quadrant B–H characteristics of permanent magnets

• Air gap line

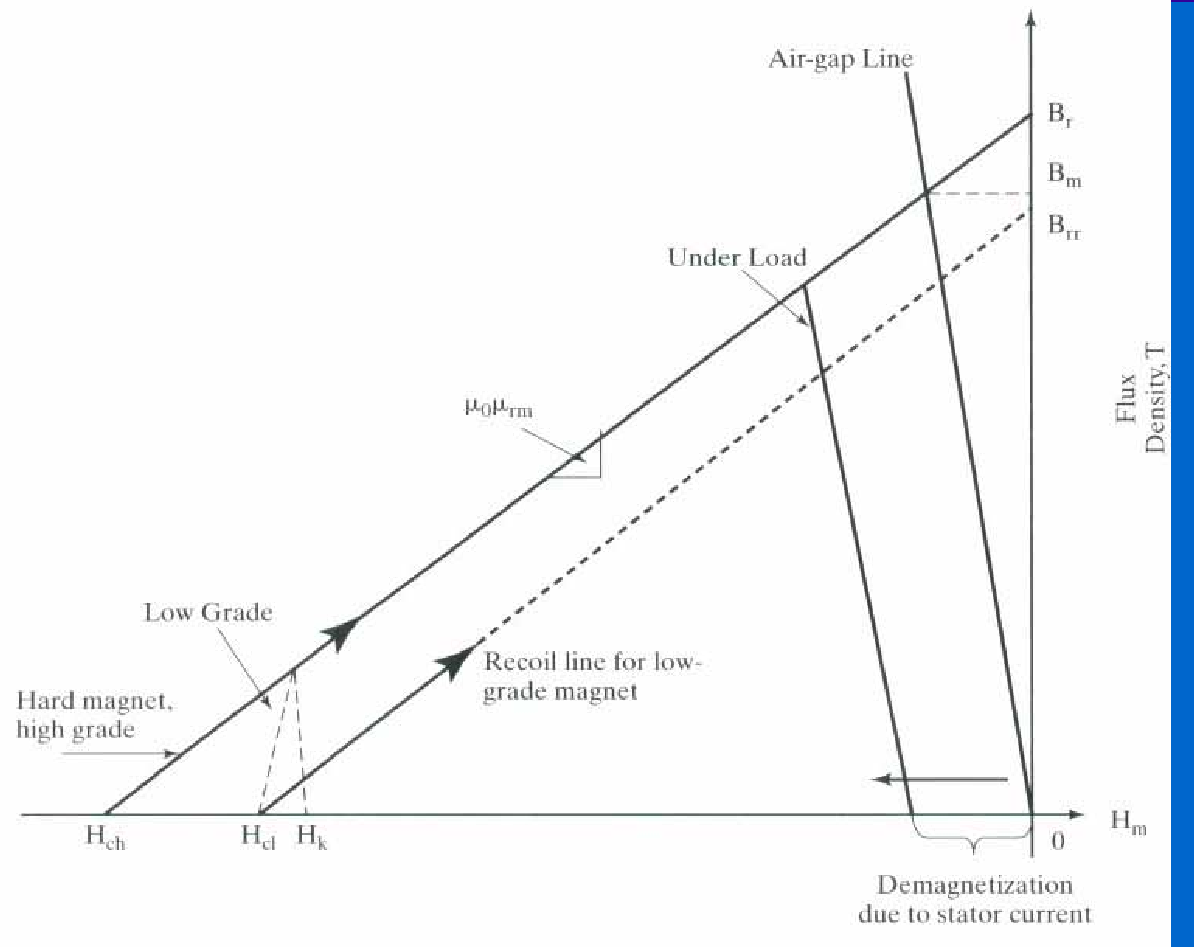


Figure 9.2 Operating point of magnets

• The flux path

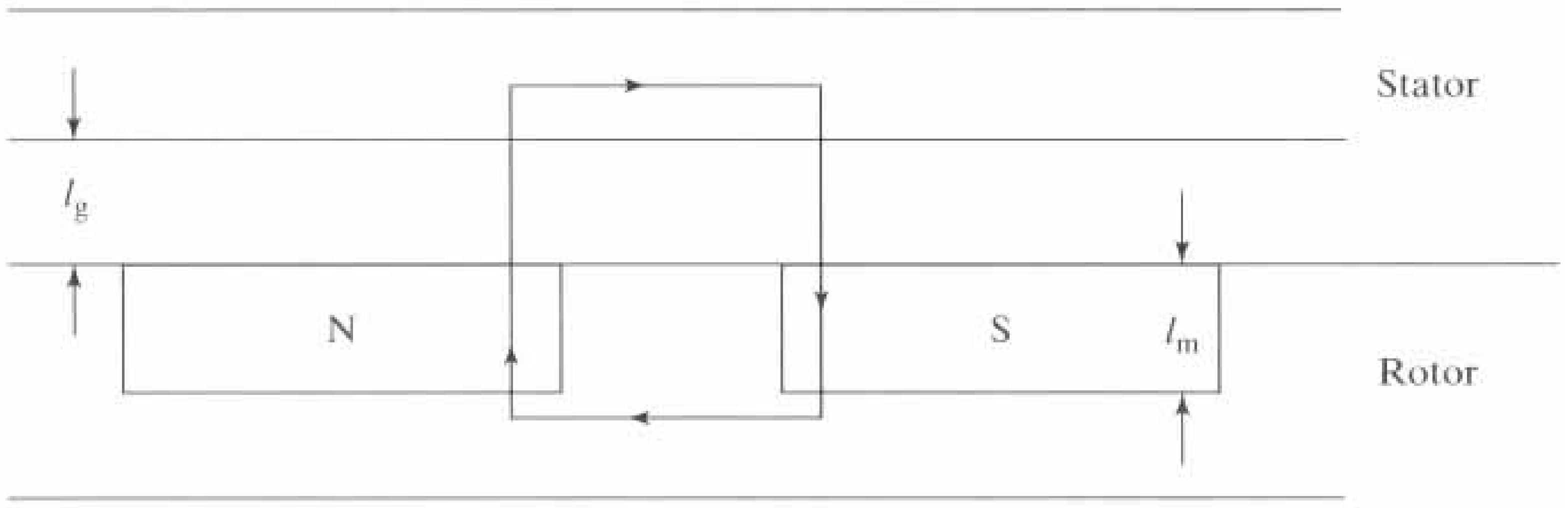


Figure 9.3 Simple layout of the stator and rotor of a machine

- If the mmf requirement of stator and rotor iron is considered negligible. Then

  $H_m l_m + H_g l_g = 0$

- The operating flux density on the demagnetization characteristic

  $B_m = B_r + \mu_0 \mu_{rm} H_m$

- $B_m = B_r + \mu_0 H_g \mu_{rm}(-l_g/l_m) = B_r - B_m \mu_{rm}(-l_g/l_m)$

$$B_m = \frac{B_r}{\left(1 + \frac{\mu_{rm} l_g}{l_m}\right)}$$

- An operating point defined by Bm and Hm

$B_m = B_r + \mu_0\mu_{rm}H_m = -\mu_0\mu_c H_m$

- The permeance coefficient is derived as

$$\mu_c = \frac{B_r}{-\mu_0 H_m} - \mu_{rm} = \frac{-\mu_0\mu_{re}H_m}{-\mu_0 H_m} - \mu_{rm} = \mu_{re} - \mu_{rm}$$

- The variations in the remnant flux density are due to temperature changes and to the impact of the applied magnetic field intensity (μ_c)
- In hard permanent magnets, the external permeability is on the order of from 1 to 10.

- The maximum energy density for a hard high grade permanent magnet is

$$E_{max} = -\frac{B_r^2}{4\mu_0\mu_{rm}}$$

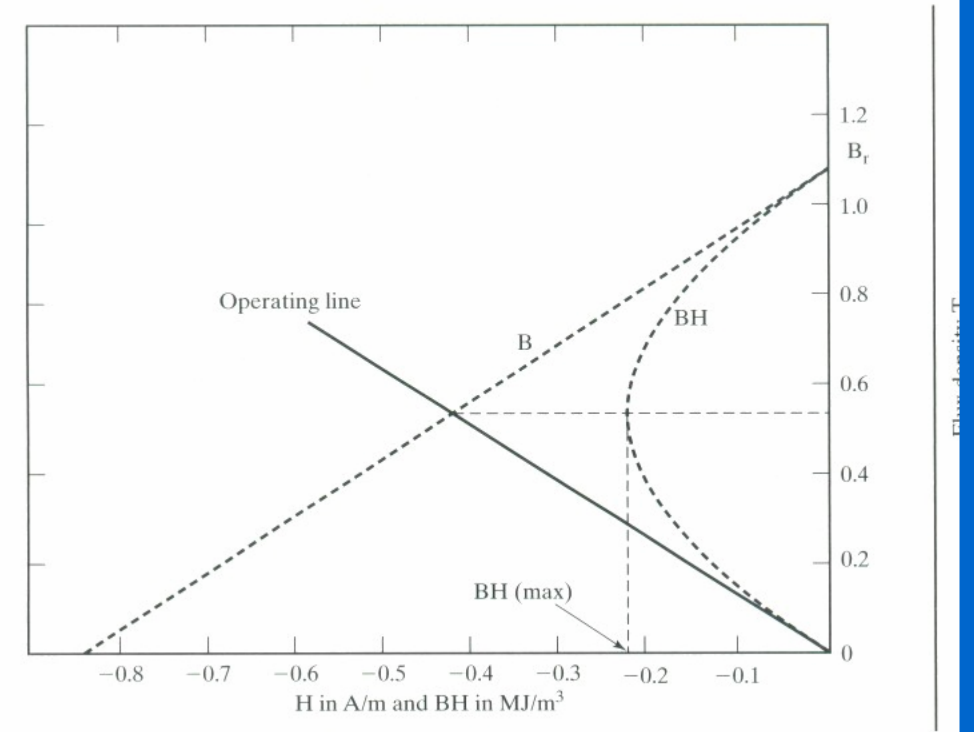


Figure 9.4 Energy product and preferred operating characteristics

- The flux density at the maximum energy density available is at $0.5B_r$.
- The magnet volume

$$B_g l_g = \mu_0 H_m l_m$$

$$B_m A_m = B_g A_g$$

- From these ideal relationships, magnet volume is

$$V_m = A_m l_m = \left(\frac{B_g A_g}{B_m}\right)\left(\frac{B_g l_g}{\mu_0 |H_m|}\right) = \frac{B_g^2 (A_g l_g)}{\mu_0 |B_m H_m|} = \frac{B_g^2 V_g}{\mu_0 |E_m|}$$

- The maximum operating energy density point of the magnet will yield the magnet with the minimum volume: cost.

9.3 Synchronous machines with PMs

- Classified on the direction of field flux

1. Radial field
2. Axial field

- Flux-density distribution (flux plot)

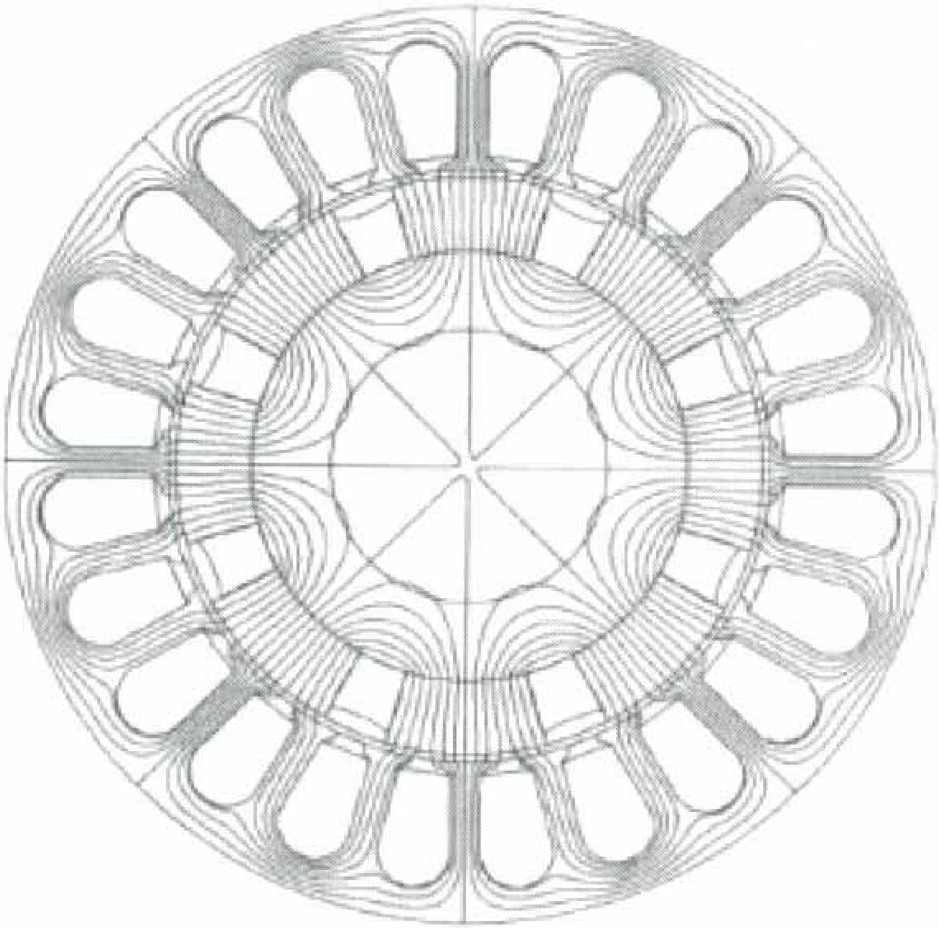


Figure 9.6 Flux distribution in the machine at no-load

• Flux-density distribution (flux density)

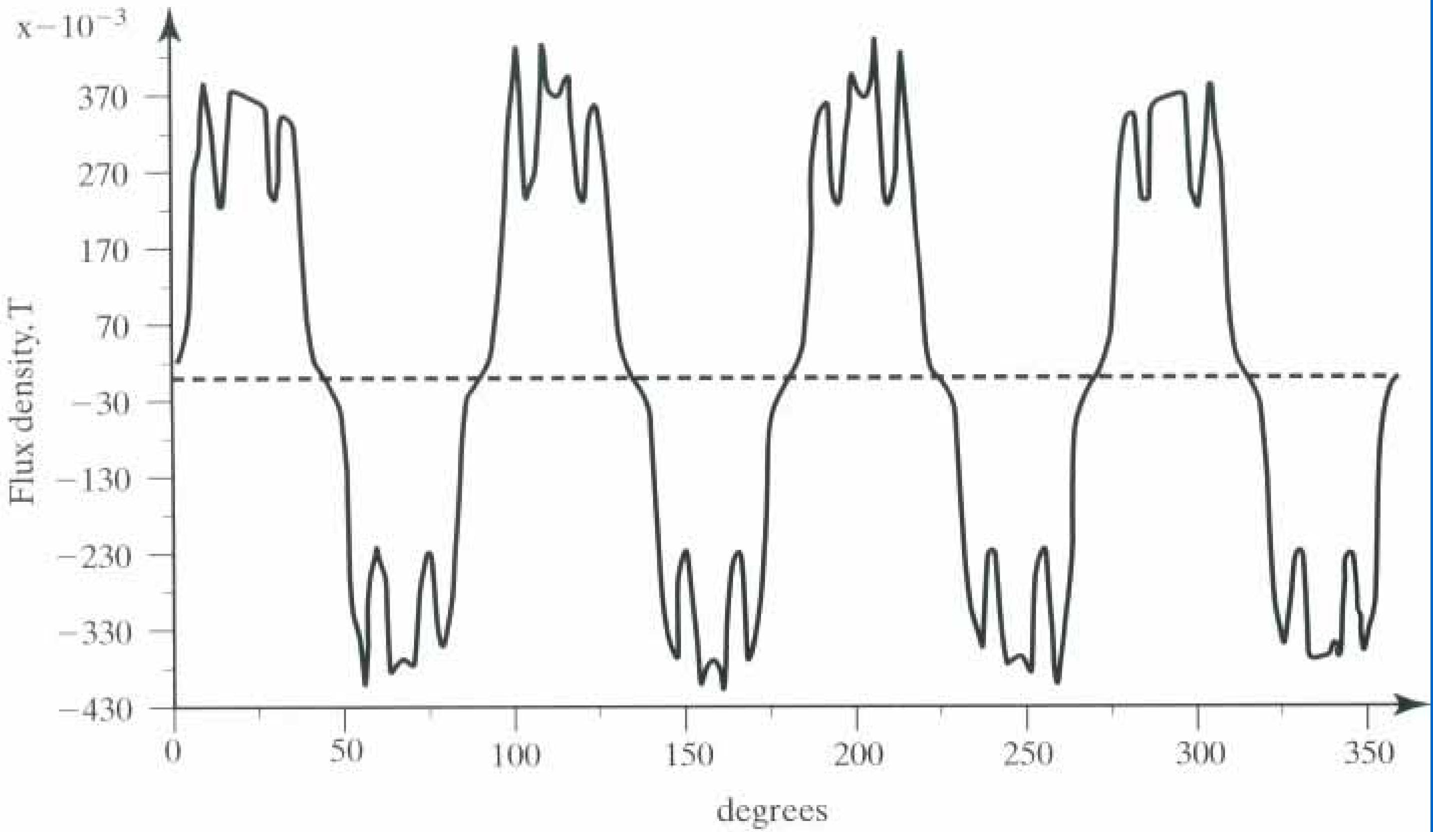


Figure 9.7 Radial air-gap flux-density profile in the machine at no-load

- Type of PM Synchronous Machines

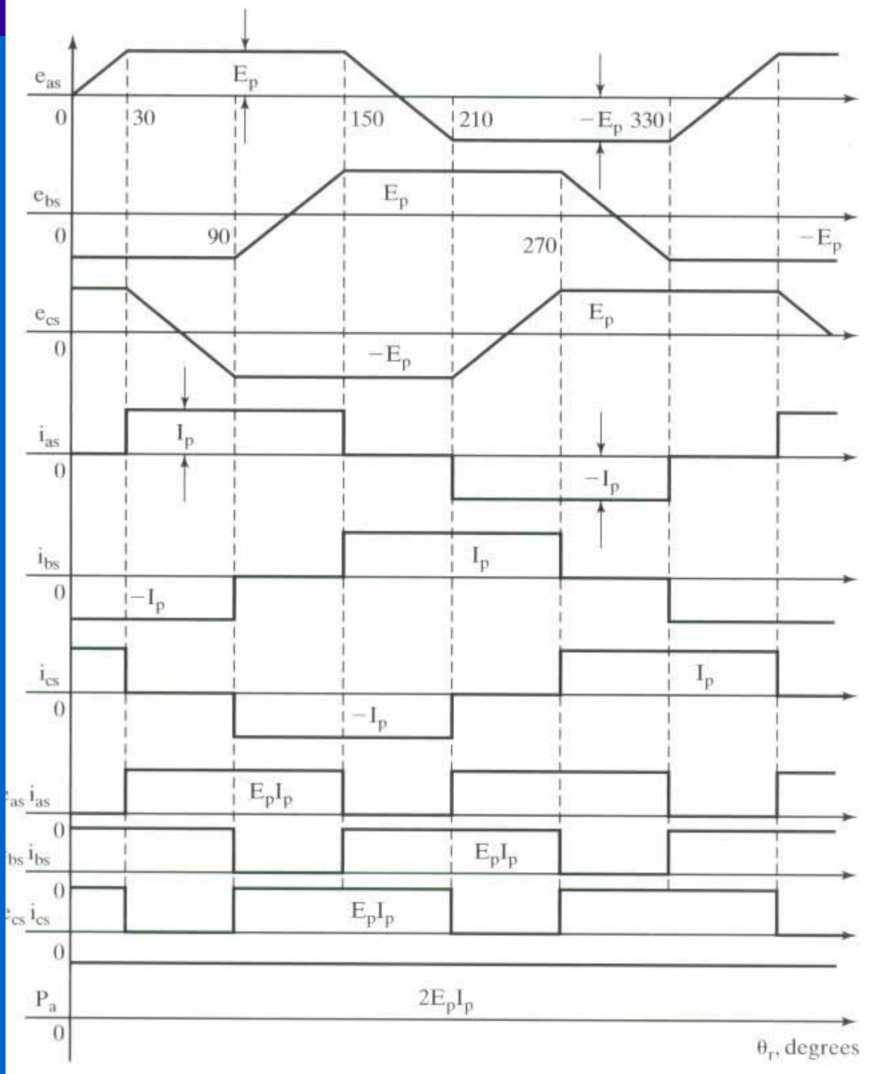


Figure 9.8 PM dc brushless-motor waveforms

- Let Ips and Ip be the peak values

$$I_{sy} = \frac{I_{ps}}{\sqrt{2}} \quad \text{(in the synchronous)}$$

$$I_d = I_p\sqrt{\frac{2}{3}} \quad \text{(in DC brushless machine)}$$

- Equating the copper losses

$$3\left(\frac{I_{ps}}{\sqrt{2}}\right)^2 R_a = 3\left(\sqrt{\frac{2}{3}} I_p\right)^2 R_a \Rightarrow I_p = \frac{\sqrt{3}}{2} I_{ps}$$

- The ratio of their power outputs

$$\text{Power output ratio} = \frac{2 \times E_p \times I_p}{3 \times \frac{E_p}{\sqrt{2}} \times \frac{I_{ps}}{\sqrt{2}}} = \frac{2 \times E_p \times \frac{\sqrt{3}}{2} I_{ps}}{3 \times \frac{E_p \times I_{ps}}{2}} = 1.1547$$

9.4 Vector control pf PMSM

- Model of the PMSM

The stator flux-linkage equations

$$v_{qs}^{r} = R_q i_{qs}^{r} + p\lambda_{qs}^{r} + \omega_r \lambda_{ds}^{r}$$

$$v_{ds}^{r} = R_d i_{ds}^{r} + p\lambda_{ds}^{r} - \omega_r \lambda_{qs}^{r}$$

where

$$\lambda_{qs}^{r} = L_s i_{qs}^{r} + L_m i_{qr}^{r}$$

$$\lambda_{ds}^{r} = L_s i_{ds}^{r} + L_m i_{dr}^{r}$$

- The q axis winding faces the interpolar path in the rotor, where the flux path encounters no magnet.
- Then the flux linkage

$$\lambda_{qs}^{r} = L_q i_{qs}^{r}$$

$$\lambda_{ds}^{r} = L_d i_{ds}^{r} + L_m i_{fr}$$

- The stator equations:

$$\begin{bmatrix} v_{qs}^{r} \\ v_{ds}^{r} \end{bmatrix} = \begin{bmatrix} R_p + L_q p & \omega_r L_d \\ -\omega_r L_q & R_d + L_d p \end{bmatrix} \begin{bmatrix} i_{qs}^{r} \\ i_{ds}^{r} \end{bmatrix} + \begin{bmatrix} \omega_r L_m i_{fr} \\ 0 \end{bmatrix}$$

- The electromagnetic torque

$$T_e = \frac{3}{2}\frac{P}{2}\{\lambda^r_{ds} i^r_{qs} - \lambda^r_{qs} i^r_{ds}\} = \frac{3}{2}\frac{P}{2}\{\lambda_{af} i^r_{qs} + (L_d - L_q) i^r_{qs} i^r_{ds}\}$$

where the rotor flux linkage linking the stator

$$\lambda_{af} = L_m i_{fr}$$

- The vector control of the PMSM is derived from its dynamic model.
- Consider the inputs, three phase current

$$i_{as} = i_s \sin(w_r t + \delta)$$

$$i_{bs} = i_s \sin(w_r t + \delta - \frac{2\pi}{3})$$

$$i_{cs} = i_s \sin(w_r t + \delta + \frac{2\pi}{3})$$

- The q and d axes stator currents in the rotor reference frame are

$$\begin{bmatrix} i_{qs}^{r} \\ i_{ds}^{r} \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \cos w_r t & \cos(w_r t - \frac{2\pi}{3}) & \cos(w_r t + \frac{2\pi}{3}) \\ \sin w_r t & \sin(w_r t - \frac{2\pi}{3}) & \sin(w_r t + \frac{2\pi}{3}) \end{bmatrix}\begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix}$$

- Substituting i_{as}, i_{bs} and i_{cs} into the above equation gives

$$\begin{bmatrix} i_{qs}^{r} \\ i_{ds}^{r} \end{bmatrix} = i_s \begin{bmatrix} \sin\delta \\ \cos\delta \end{bmatrix}$$

- The q and d axes currents are constant, since δ is a constant for a given load torque.

- The electromagnetic torque then becomes

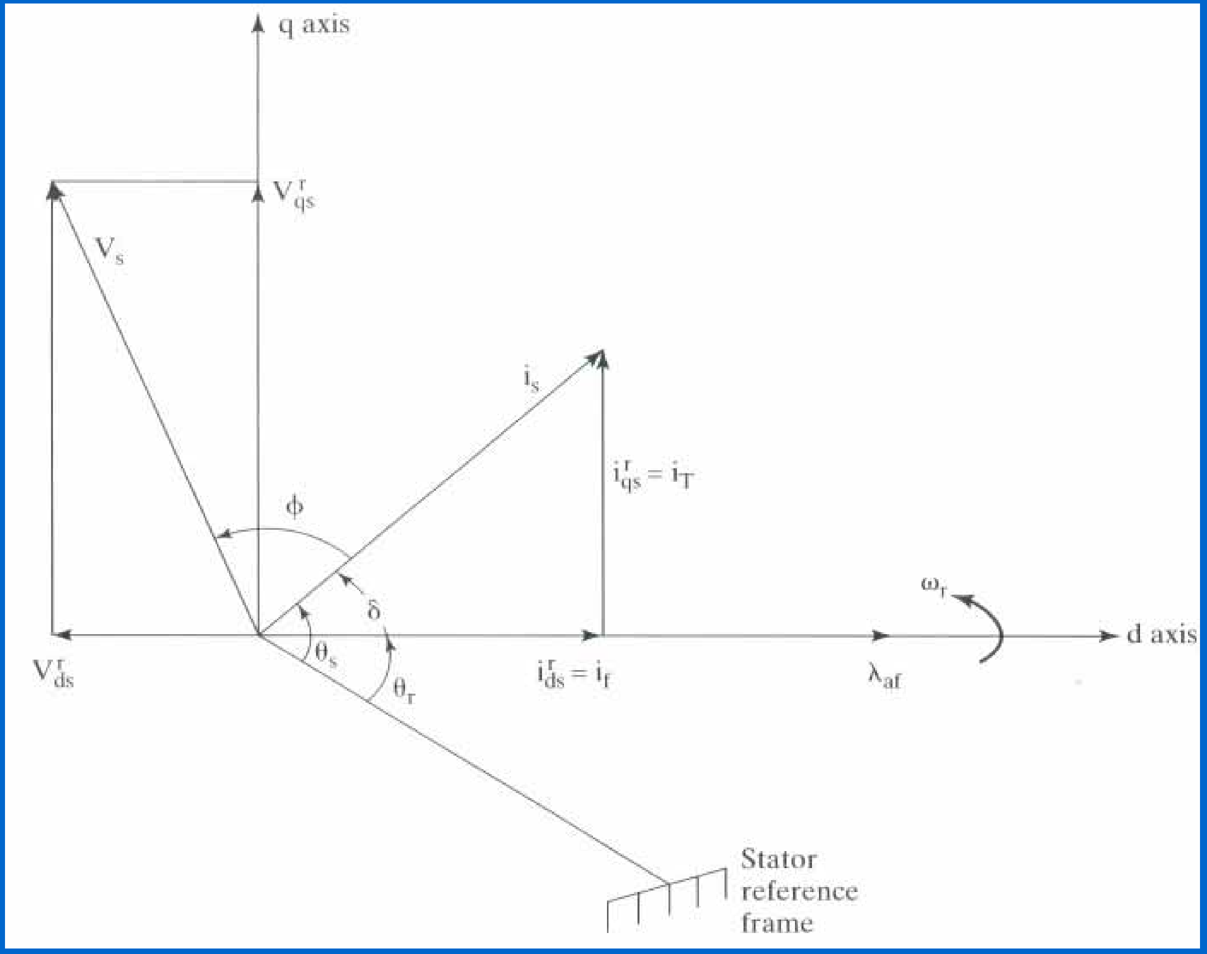
$$T_e = \frac{3}{2}\frac{P}{2}\{(L_d - L_q)i_{qs}^r i_{ds}^r + \lambda_{af} i_{ds}^r\} = \frac{3}{2}\frac{P}{2}[\frac{1}{2}(L_d - L_q)i_s^2 \sin 2\delta + \lambda_{af} i_s \sin\delta]$$

- For $\delta = \pi/2$,

$$T_e = \frac{3}{2}\frac{P}{2}\lambda_{af} i_s = K_1 \lambda_{af} i_s, \quad (N \cdot m)$$

- The above equation is similar to that of the torque generated in the dc motor.
- If the torque angle is maintained at 90º and flux is kept constant, then the torque is controlled by stator current.

- The phasor diagram for an arbitrary torque angle δ:



- Torque-producing component of stator current = $i_T = i^r_{qs}$
- Flux-producing component of stator current = $i_f = i^r_{ds}$
- The torque angle $\delta = \theta_T$.
- The mutual flux linkage

$$\lambda_m = \sqrt{(\lambda_{af} + L_d i^r_{ds})^2 + (L_q i^r_{qs})^2}$$

• Motor drive schematic

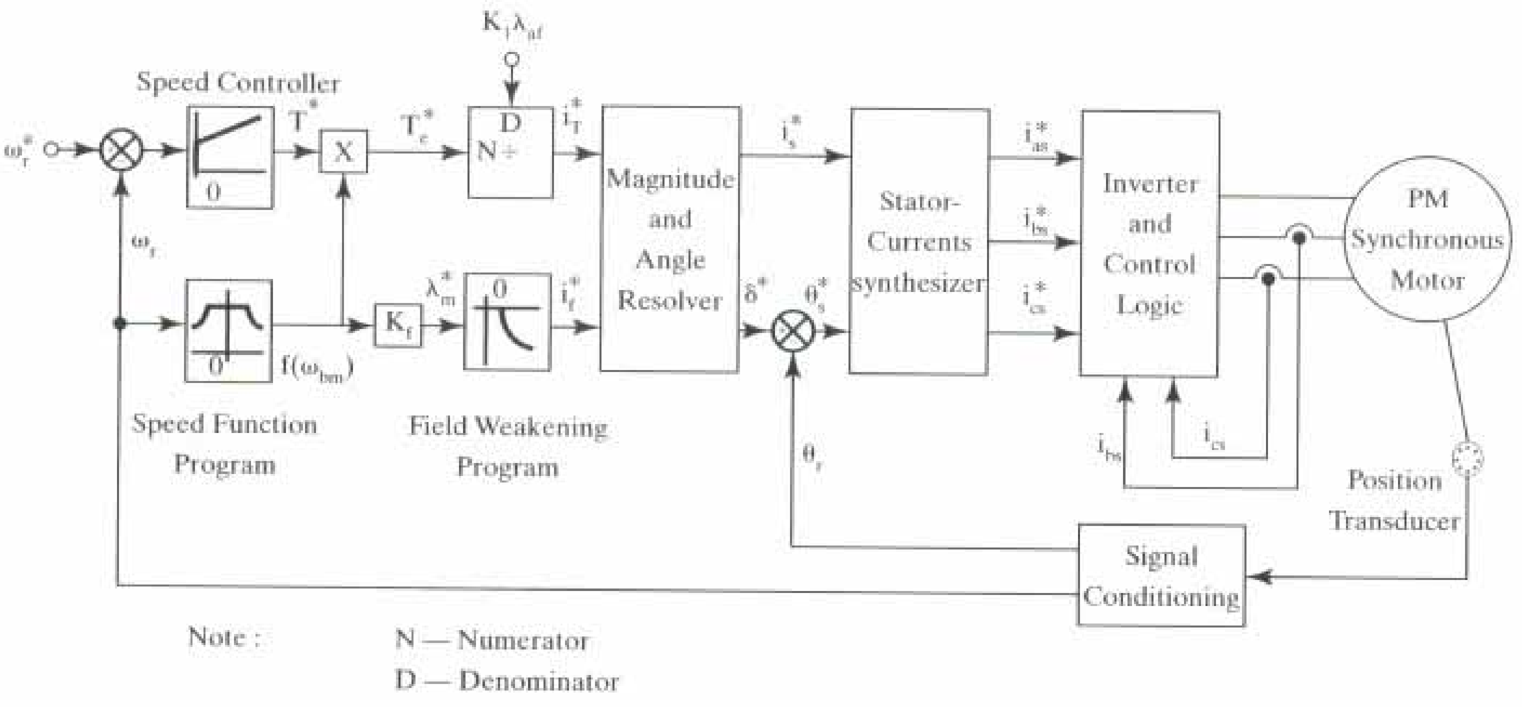


Figure 9.10 Vector-controlled PM synchronous motor drive

- The function generator on the speed

$$f(\omega_{bm}) = \begin{cases} \frac{\omega_b}{\omega_m}; & \pm\omega_b < \omega_m < \pm\omega_{max} \\ 1; & 0 < \omega_m < \pm\omega \end{cases}$$

- The air gap power

$$P_a = \omega_m T_e^* = \omega_m f(\omega_{bm}) T^* = \begin{cases} \omega_m T^* & \text{for the constant - torque region} \\ \omega_b T^* & \text{for the constant - power region} \end{cases}$$

- The stator-current phasor magnitude and the torque angle

$$i_s^* = \sqrt{(i_f^*)^2 + (i_T^*)^2}$$

$$\delta^* = \tan^{-1}\left[\frac{i_T^*}{i_f^*}\right]$$

- The instantaneous position of the stator-current command

$$\theta_s^* = \theta_r + \delta^* = \omega_r t + \delta^*$$

- The stator-phase current commands

$$\begin{bmatrix} i_{as}^* \\ i_{bs}^* \\ i_{cs}^* \end{bmatrix} = \begin{bmatrix} \cos\theta_r & \sin\theta_r \\ \cos(\theta_r - \frac{2\pi}{3}) & \sin(\theta_r - \frac{2\pi}{3}) \\ \cos(\theta_r + \frac{2\pi}{3}) & \sin(\theta_r + \frac{2\pi}{3}) \end{bmatrix} \begin{bmatrix} i_T^* \\ i_f^* \end{bmatrix} = i_s^* \begin{bmatrix} \sin(\theta_r + \delta^*) \\ \sin(\theta_r + \delta^* - \frac{2\pi}{3}) \\ \sin(\theta_r + \delta^* + \frac{2\pi}{3}) \end{bmatrix}$$

9.5 Control strategies

- Some key control strategies

(i) constant torque angle control or zero-direct-axis-current control

(ii) unity power-factor control

(iii) constant mutual air gap flux-linkage control

(iv) optimum-torque-per-ampere control

(v) flux-weakening control

9.7 Speed-controller design

- The motor q axis voltage equation

$$v_{qs}^{r} = (R_s + L_q p)i_{qs}^{r} + \omega_r \lambda_{af}$$

- The electromagnetic equation

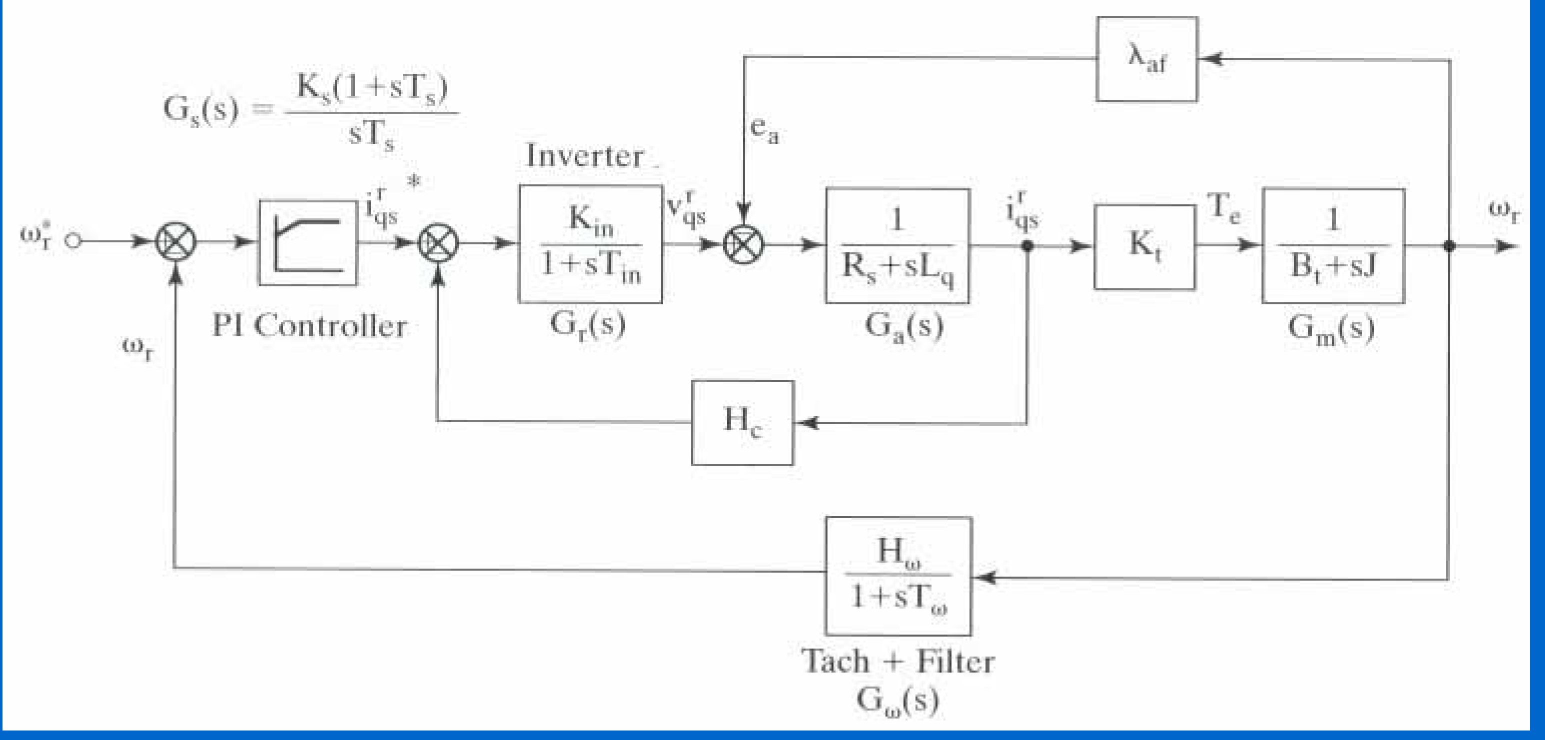
$$\frac{P}{2}(T_e - T_l) = Jp\omega_r + B_l\omega_r$$

where $T_e = \frac{3}{2} \cdot \frac{P}{2} \lambda_{af} i_{qs}^{r}$

- If $T_l = B_l\omega_r$, then

$$(Jp + B_t)\omega_r = \{\frac{3}{2}\cdot(\frac{P}{2})^2\lambda_{af}\}i_{qs}^r = K_t i_{qs}^r$$

where $B_t = \frac{P}{2}B_l + B_1 \quad K_t = \frac{3}{2}(\frac{P}{2})^2\cdot\lambda_{af}$



9.8 sensorless control

- The following assumption are made

1 Motor parameters and rotor PM flux are constant

2 Induced emfs in the machine are sinusoidal

3 The drive operates in the constant-torque region, and flux-weakening operation is not considered.

• The basic control schematic diagram

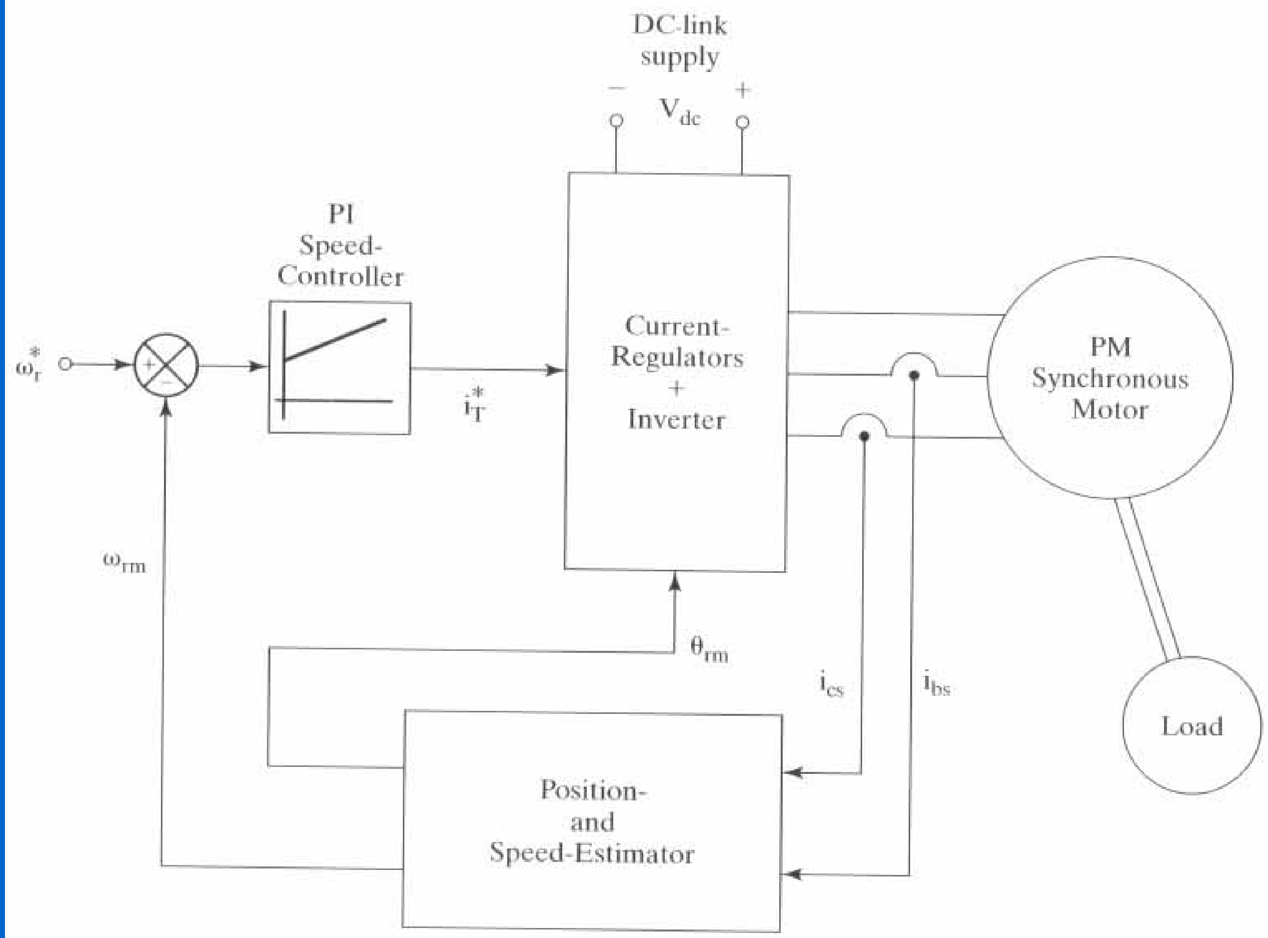


Figure 9.38 Control schematic diagram of the sensorless PMSM drive

• Phasor diagram corresponding to an error

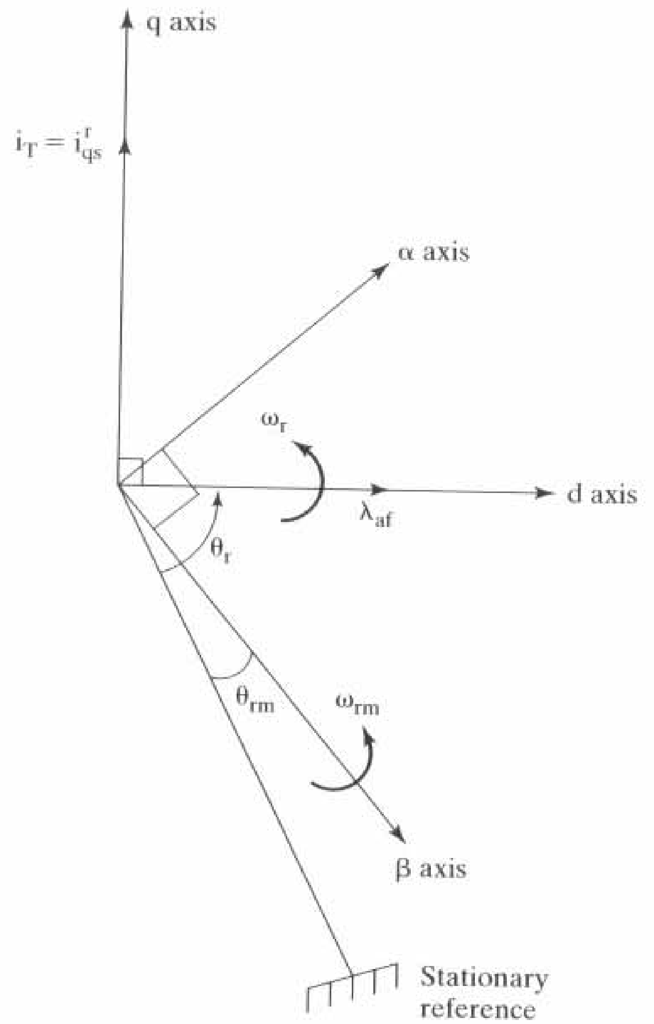


Figure 9.39 Phasor diagram corresponding to an error between the actual and assumed rotor position

- Assume that the rotor position θ_{rm} lags behind the actual rotor position θ_r by $\delta\theta$ radian

$$\theta_r = \int \omega_r dt$$

$$\theta_{rm} = \int \omega_{rm} dt$$

$$\delta\theta = \theta_r - \theta_{rm} = \int (\omega_r - \omega_{rm}) dt$$

- The stator current carried out in a reference frame at an assumed rotor speed.
- The reference frames are α and β axes.

- The machine equations in rotor-speed reference

$$\begin{bmatrix} pi_{\alpha m} \\ pi_{\beta m} \end{bmatrix} = \begin{bmatrix} -\frac{R_s}{L_q} & -\omega_{rm}\frac{L_d}{L_q} \\ \omega_{rm}\frac{L_q}{L_d} & -\frac{R_s}{L_q} \end{bmatrix} \begin{bmatrix} i_{\alpha m} \\ i_{\beta m} \end{bmatrix} + \begin{bmatrix} -\frac{\omega_{rm}\lambda_{af}}{L_q} \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{v_\alpha}{L_q} \\ \frac{v_b}{L_d} \end{bmatrix}$$

- The actual machine equations in d and q axes

$$\begin{bmatrix} pi_{\alpha} \\ pi_{\beta} \end{bmatrix} = \begin{bmatrix} -\frac{R_s}{L_q} & -\omega_{rm}\frac{L_d}{L_q} \\ \omega_{rm}\frac{L_q}{L_d} & -\frac{R_s}{L_q} \end{bmatrix} \begin{bmatrix} i_{\alpha} \\ i_{\beta} \end{bmatrix} + \begin{bmatrix} -\frac{\omega_r\lambda_{af}}{L_q}\cos\delta\theta \\ \frac{\omega_r\lambda_{af}}{L_q}\sin\delta\theta \end{bmatrix} + \begin{bmatrix} \frac{v_\alpha}{L_q} \\ \frac{v_b}{L_d} \end{bmatrix}$$

$$\text{using } \begin{bmatrix} \cos\delta\theta & \sin\delta\theta \\ -\sin\delta\theta & \cos\delta\theta \end{bmatrix}$$

- Discretize the two sets of model and actual

$$\begin{bmatrix} i_{\alpha m}[kT] \\ i_{\beta m}[kT] \end{bmatrix} = \begin{bmatrix} i_{\alpha m}[(k-1)T] \\ i_{\beta m}[(k-1)T] \end{bmatrix} + \begin{bmatrix} pi_{\alpha m}[(k-1)T] \\ pi_{\beta m}[(k-1)T] \end{bmatrix} T$$

$$\begin{bmatrix} i_{\alpha}[kT] \\ i_{\beta}[kT] \end{bmatrix} = \begin{bmatrix} i_{\alpha}[(k-1)T] \\ i_{\beta}[(k-1)T] \end{bmatrix} + \begin{bmatrix} pi_{\alpha}[(k-1)T] \\ pi_{\beta}[(k-1)T] \end{bmatrix} T$$

- The current errors

$$\begin{bmatrix} \delta i_{\alpha}[kT] \\ \delta i_{\beta}[kT] \end{bmatrix} = \begin{bmatrix} i_{\alpha}[kT] - i_{\alpha m}[kT] \\ i_{\beta}[kT] - i_{\beta m}[kT] \end{bmatrix} = T \begin{bmatrix} -\frac{\lambda_{af}}{L_q}(\omega_r \cos\delta\theta - \omega_{rm}) \\ \frac{\omega_r \lambda_{af}}{L_d} \sin\delta\theta \end{bmatrix}$$

- If $\delta\theta$ is small

$$\begin{bmatrix} \delta i_{\alpha}[kT] \\ \delta i_{\beta}[kT] \end{bmatrix} = T \begin{bmatrix} -\frac{\lambda_{af}}{L_q}(-\omega_{rm} + \omega_r) \\ \frac{\omega_r \lambda_{af}}{L_d} \delta\theta \end{bmatrix}$$

- The actual rotor speed is obtained as

$$\omega_r = -\frac{L_q}{\lambda_{af}}\frac{1}{T}\delta i_\alpha[kT] + \omega_{rm}$$

- The error rotor-speed

$$\delta\theta = \frac{L_d}{\lambda_{af}}\frac{1}{T}\frac{\delta i_\beta[kT]}{\omega_r} = \frac{\left(\frac{L_d}{T\lambda_{af}}\right)\delta i_\beta[kT]}{\left[\omega_{rm} - \frac{L_q}{T\lambda_{af}}\right]\delta i_\alpha[kT]}$$

- The rotor position then is

$$\theta_r = \theta_{rm} + \delta\theta$$

• Block-diagram realization

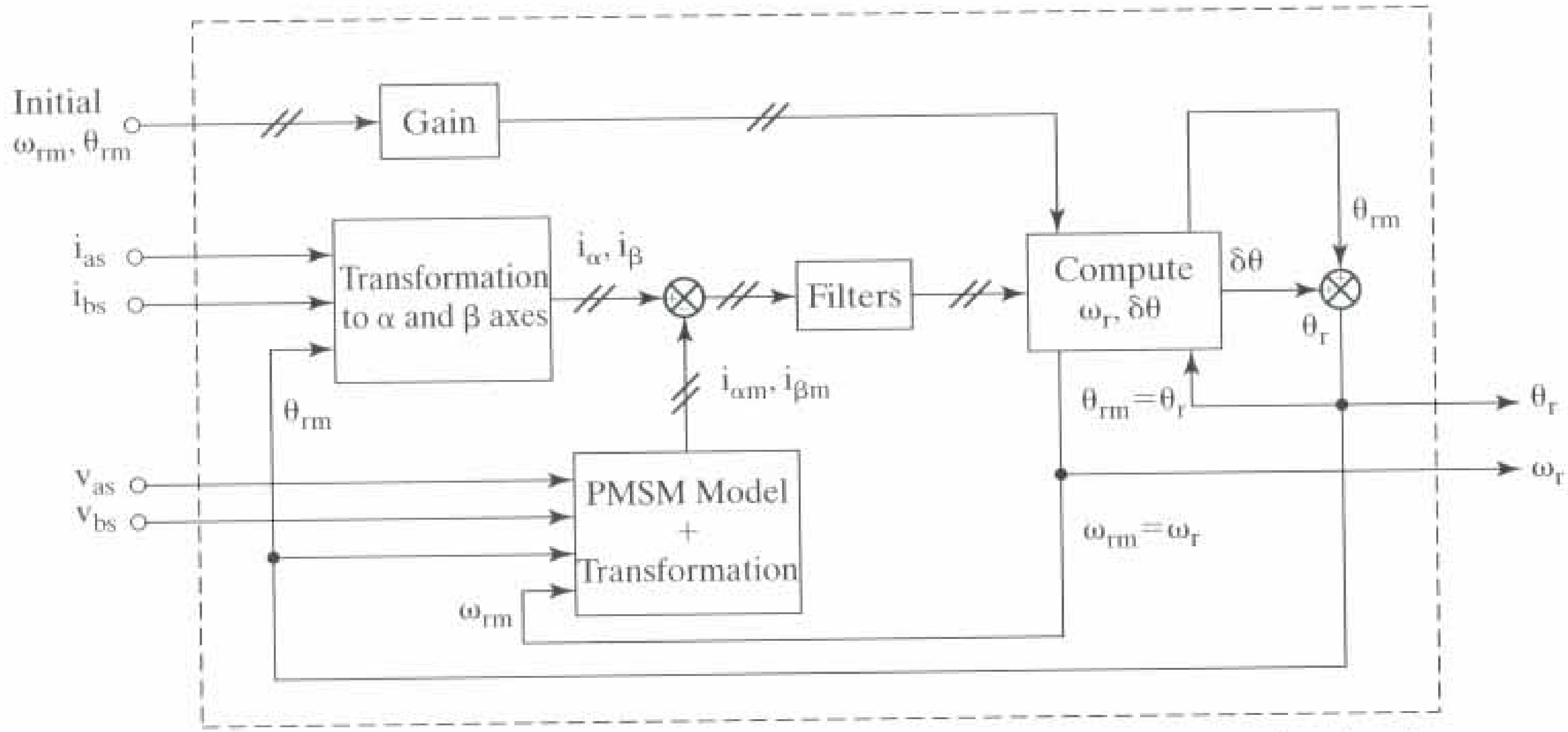


Figure 9.40 Block-diagram realization of electrical rotor position and speed estimation

9.10 PM Brushless DC Motor

- The couple circuit equations

$$\begin{bmatrix} v_{as} \\ v_{bs} \\ v_{cs} \end{bmatrix} = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} + p \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} + \begin{bmatrix} e_{as} \\ e_{bs} \\ e_{cs} \end{bmatrix}$$

$$\begin{bmatrix} v_{as} \\ v_{bs} \\ v_{cs} \end{bmatrix} = R_s \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} + \begin{bmatrix} L & M & M \\ M & L & M \\ M & M & L \end{bmatrix} p \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} + \begin{bmatrix} e_{as} \\ e_{bs} \\ e_{cs} \end{bmatrix}$$

$$\begin{bmatrix} v_{as} \\ v_{bs} \\ v_{cs} \end{bmatrix} = R_s \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} + \begin{bmatrix} (L-M) & 0 & 0 \\ 0 & (L-M) & 0 \\ 0 & 0 & (L-M) \end{bmatrix} p \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} + \begin{bmatrix} e_{as} \\ e_{bs} \\ e_{cs} \end{bmatrix}$$

- The peak value of induced emfs E_p

$$E_p = (Blv)N = N(Blr\omega_m) = N\phi_a\omega_m = \lambda_p\omega_m$$

- The air gap flux ϕ_g

$$\phi_a = Blr = \frac{1}{\pi}B\pi lr = \frac{1}{\pi}\phi_g$$

- The instantaneous induced emfs

$$e_{as} = f_{as}(\theta_r)\lambda_p\omega_m$$

$$e_{bs} = f_{bs}(\theta_r)\lambda_p\omega_m$$

$$e_{bs} = f_{bs}(\theta_r)\lambda_p\omega_m$$

- The electromagnetic torque

$T_e = [e_{as}i_{as} + e_{bs}i_{bs} + e_{cs}i_{cs}]/\omega_m$

$= \lambda_p[f_{as}(\theta_r)i_{as} + f_{bs}(\theta_r)i_{bs} + f_{bs}(\theta_r)i_{cs}]$

- The motion equation

$$J\frac{d\omega_m}{dt} + B\omega_m = (T_e - T_l)$$

- The relationship between electrical rotor speed and position

$$\frac{d\theta_r}{dt} = \frac{P}{2}\omega_m$$

- The system in state-space form

$$\dot{x} = Ax + Bu$$

where $L_1 = L - M$, $x = [i_{as}\ i_{bs}\ i_{cs}\ \omega_m\ \theta_r]^t$

$u = [v_{as}\ v_{bs}\ v_{cs}\ T_l]^t$

$$A = \begin{bmatrix} -\frac{R_s}{L_1} & 0 & 0 & -\frac{\lambda_p}{L_1} f_{as}(\theta_r) & 0 \\ 0 & -\frac{R_s}{L_1} & 0 & -\frac{\lambda_p}{L_1} f_{bs}(\theta_r) & 0 \\ 0 & 0 & -\frac{R_s}{L_1} & -\frac{\lambda_p}{L_1} f_{cs}(\theta_r) & 0 \\ \frac{\lambda_p}{J} f_{as}(\theta_r) & \frac{\lambda_p}{J} f_{as}(\theta_r) & \frac{\lambda_p}{J} f_{as}(\theta_r) & -B/J & 0 \\ 0 & 0 & 0 & \frac{P}{2} & 0 \end{bmatrix} \qquad B = \begin{bmatrix} \frac{1}{L_1} & 0 & 0 & 0 \\ 0 & \frac{1}{L_1} & 0 & 0 \\ 0 & 0 & \frac{1}{L_1} & 0 \\ 0 & 0 & 0 & -\frac{1}{J} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

- Speed-controlled PMBDM drive scheme