4 Chopper-Controlled DC Motor Drive

• Chopper: The variable dc voltage is controlled by varying the on- and off-times of a converter.

• Fig. 4.1 is a schematic diagram of the chopper.
• Its frequency of operation is

\[ f_c = \frac{1}{(t_{on} + t_{off})} = \frac{1}{T} \]

and its duty cycle is defined as

\[ d = \frac{t_{on}}{T} \]

• Assuming that the switch is ideal, the average output is

\[ V_{dc} = \frac{t_{on}}{T} V_s = dV_s \]

• Varying the duty cycle changes the output voltage.
The duty cycle $d$ can be changed in two ways:

(i) varying the on-time (constant switching frequency).

(ii) varying the chopping frequency.

Constant switching frequency has many advantages in practice.
4.3 Four-Quadrant Chopper Circuit

- Fig. 4.2 is the chopper circuit with transistor switches.

![Figure 4.2: A four-quadrant chopper circuit](image)
• Fig. 4.3 is first-quadrant operation.

Figure 4.3  First-quadrant operation with positive voltage and current in the load
- Fig. 4.4 is first-quadrant operation with zero voltage across the load.

Figure 4.4  First-quadrant operation with zero voltage across the load
Fig. 4.5 voltage and current waveforms in first-quadrant operation.
• Fig. 4.6 second-quadrant operation

**Figure 4.6** Second-quadrant operation, with negative load voltage and positive current
• Fig. 4.7 voltage and current waveforms in second-quadrant operation.
• Fig. 4.8 Modes of operation in the third quadrant.
• **Fig. 4.9 waveform in third-quadrant operation**

![Waveform Diagram](image-url)

**Figure 4.9** Third-quadrant operation
• Fig. 4.10 Waveforms in fourth-quadrant operation.
4.4 Chopper for inversion

- Converter: DC ⇒ DC (different voltage)
- The chopper is the building block for that: AC ⇒ DC ⇒ DC (different voltage)
4.5 Chopper with other power devices

- MOSFETs, IGBTs, GTOs, or SCRs are used for different power level.
- The MOSFET and transistor choppers are used at power levels up to 50kW.
4.6 Model of The Chopper

- The transfer function of the chopper is

$$G_r(s) = \frac{K_r}{1 + sT}$$

where $K_r = V_s/V_{cm}$, $V_s$ is the source voltage, and $V_{cm}$ is the maximum control voltage.

- Increasing the chopping frequency decreases the delay time, and its becomes a simple gain.
4.7 Input to the chopper

- The chopper input: a battery or a rectified ac supply.
- Fig. 4.11 is the chopper with rectified circuit.
• Its disadvantage: it cannot transfer power from dc link into ac mains.

• Fig. 4.12 Chopper with regeneration capability
• The generating converter has to be operated at triggering angles greater than 90 °.
4.8 Other Chopper Circuits

- Fig. 4.13 one- and two-quadrant operation.
4.9 Steady-state analysis ...

• 4.9.1 Analysis by averaging
• The average armature current is
  \[ I_{av} = \frac{V_{dc} - E}{R_a} \]
  where \( V_{dc} = dV_s \)
• The electromagnetic torque is
  \[ T_{av} = K_b I_{av} = K_b(dV_s-K_bw_m)/R_a \text{ (N} \cdot \text{m}) \]
The normalized torque is

\[ T_{en} = \frac{T_{av}}{V_b} = \frac{K_b(dV_s - K_b\omega_m)}{V_b} = \frac{dV_n - \omega_{mn}}{R_{an}}, \text{ p.u.} \]

where \( R_{an} = \frac{I_b R_a}{V_b}, \text{ p.u.} \), \( V_n = \frac{V_s}{V_b}, \text{ p.u.} \), \( \omega_{mn} = \frac{\omega_m}{\omega_b}, \text{ p.u.} \)

4.9.2 Instantaneous steady-state computation (including harmonics)

The equations of the motor for on and off times (continuous current conduction)

\[ V_s = E + R_a i_a + L_a \frac{di_a}{dt}, \quad 0 \leq t \leq dT \]

\[ 0 = E + R_a i_a + L_a \frac{di_a}{dt}, \quad dT \leq t \leq T \]
Fig. 4.15 The waveform of applied voltage and armature current.

Figure 4.15  Applied voltage and armature current in a chopper-controlled dc motor drive
• The solutions of the above equations:

\[ i_a(t) = \frac{V_s - E}{R_a}(1 - e^{-\frac{t}{T_a}}) + I_{a0}e^{-\frac{t}{T_a}}, \quad 0 < t < dT \]

\[ i_a(t) = -\frac{E}{R_a}(1 - e^{-\frac{t^1}{T_a}}) + I_{a1}e^{-\frac{t^1}{T_a}}, \quad dT \leq t \leq dT \]

where \( T_a = L_a/R_a \) (Armature time constant)

\[ t^1 = t - dT \]

• By using boundary condition

\[ I_{a1} = \frac{V_s(1 - e^{-dT/T_a})}{R_a(1 - e^{-T/T_a})} - \frac{E}{R_a} \]

\[ I_{a0} = \frac{V_s(e^{dT/T_a} - 1)}{R_a(e^{T/T_a} - 1)} - \frac{E}{R_a} \]
• The critical duty cycle: the limiting or minimum value of duty cycle for continuous current; \( I_{a0} \) equates to zero.

• The relevant equations for discontinuous current-conduction mode

\[
\begin{align*}
V_s &= E + R_a i_a + L_a \frac{di_a}{dt}, \quad 0 \leq t \leq dT \\
0 &= E + R_a i_a + L_a \frac{di_a}{dt}, \quad dT \leq t \leq t_x + dT \\
\text{with } i_a(t_x + dT) &= 0; \quad i_a(0) = 0
\end{align*}
\]
• Hence,

\[ I_{a1} = \frac{V_s - E}{R_a} (1 - e^{-dT/T_a}) \]

\[ i_a(t_x + dT) = -\frac{E}{R_a} (1 - e^{-t_x/T_a}) + I_{a1} e^{-t_x/T_a} \]

• \( t_x \) is evaluated from (4.25) and the above eqs. as

\[ t_x = T_a \log_e\left[1 + \frac{I_{a1} R_a}{E}\right] \]
The solution for the armature current in three time segments is

\[ i_a(t) = \frac{V_s - E}{R_a} \left( 1 - e^{-\frac{t}{T_a}} \right), \quad 0 < t < dT \]

\[ i_a(t) = I_{a1} e^{-\frac{(t-dT)}{T_a}} - \frac{E}{R_a} \left( 1 - e^{-\frac{(t-dT)}{T_a}} \right), \quad dT \leq t \leq t_x - dT \]

\[ i_a(t) = 0, \quad (t_x + dT) < t < T \]
4.10 Rating of the device

- The rms value of the power switch current
  \[ I_t = \sqrt{\frac{I_{\text{max}}^2}{2T} (T + dT)} = \sqrt{\frac{1 + d}{2}} \cdot I_{\text{max}} \]

- The average diode current
  \[ I_d = \left(\frac{1 - d}{2}\right) \cdot I_{\text{max}} \]
• Transistor and diode currents for motoring operation in the continuous-conduction mode.
4.11 Pulsating Torque

- The average of the harmonic torque is zero.
- High-performance applications require the pulsating torque to be minimum.
- The applied voltage is resolved into Fourier components as

\[ V_a(t) = V_a + \sum_{n=1}^{\infty} A_n \sin(n\omega_c t + \theta_n) \]

where

- \( V_a = \frac{\Delta T}{T} \)
- \( V_s = dV_s \)
- \( \omega_c = 2\pi f_c = \frac{2\pi}{T} \)
- \( A_n = \frac{2V_s}{n\pi} \sin\left(\frac{n\omega_c dT}{2}\right) \)
- \( \theta_n = \frac{\pi}{2} - \frac{n\omega_c dT}{2} \)
• The armature current is expressed as

\[ i_a(t) = I_{av} + \sum_{n=1}^{\infty} \frac{A_n}{|Z_n|} \sin(n\omega_c t + \theta_n - \phi_n) \]

where \( I_{av} = \frac{V_a - E}{R_a} \)

\[ Z_n = R_a + jn\omega_c L_a \]

\[ \phi_n = \cos^{-1}\left\{ \frac{R_a}{\sqrt{R_a^2 + n^2\omega_c^2 L_a^2}} \right\} \]

• The input power is

\[ P_i = V_a(t)i_a(t) \]

\[ = \left\{ V_a I_{av} + I_{av} \sum_{n=1}^{\infty} A_n \sin(n\omega_c t + \theta_n) \right\} \]

\[ + V_a \sum_{n=1}^{\infty} \frac{A_n}{|Z_n|} \sin(n\omega_c t + \theta_n - \phi_n) + \sum_{n=1}^{\infty} \left( \frac{A_n^2}{|Z_n|^2} \sin(n\omega_c t + \theta_n) \sin(n\omega_c t + \theta_n - \phi_n) \right) \]
4.12 Closed-loop operation

- Fig. 4.20 is the closed-loop speed-controller dc motor drive.
The current controller can be
(i) Pulse-Width-Modulation (PWM) controller
(ii) Hysteresis controller
Fig. 4.21 shows on- and off-time for PWM
Fig. 4.22 is the implementation of PWM current controller.

![Diagram of PWM current controller](image)

**Figure 4.22**  PWM current-controller implementation with ramp carrier signal

*Note: ZCD — Zero Crossing Detector for polarity detection*
Fig. 4.24 shows hysteresis-controlled operation

\[ i_a \leq i_a^* - \Delta i, \quad \text{set } v_a = V_s \]

\[ i_a \geq i_a^* + \Delta i, \quad \text{reset } v_a = 0 \]
Figure 4.23: Principle of PWM operation
• Fig. 4.25 is realization of hysteresis controller.