3 Phase-controlled DC motor drives

- 3.1 Introduction
- Two types of speed control: armature control and field control

3.2 Principles of DC motors speed Control

- The induced voltage is dependent on the field flux and speed

$$e = K\phi_f\omega_m$$

- The flux is proportional to the field current

$$\phi_f \propto i_f$$

- The speed is expressed as

$$\omega_m \propto \frac{e}{\phi_f} \propto \frac{e}{i_f} \propto \frac{(v - i_a R_a)}{i_f}$$

- The rotor speed is dependent on the applied voltage and field current.

- In field control, the applied armature voltage v is maintained constant.
- Then the speed is represented as

$$\omega_m \propto \frac{1}{i_f}$$

- In armature control, the field current is maintained constant. Then the speed is derived as

$$\omega_m \propto (v - i_a R_a)$$

- Hence, varying the applied voltage changes speed. Reversing the applied voltage changes the direction of rotation of the motor.
- The advantage: the field time constant is at least 10 to 100 times greater than the armature time constant.
- Armature control is idea for speeds lower than rated speed; field control is suitable above for speeds greater than the rated speed.

- Fig. 3.1

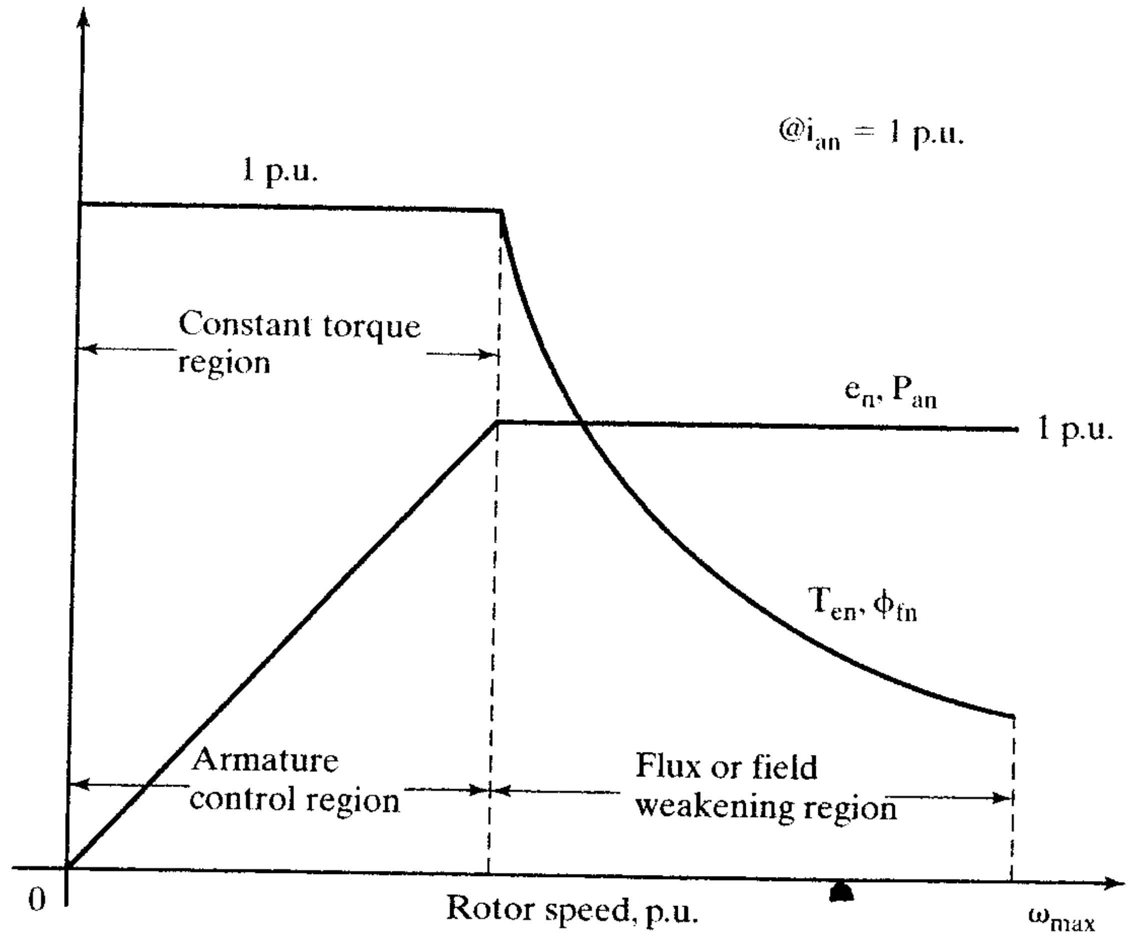


Figure 3.1 Normalized characteristics of a variable-speed dc motor

- By combining armature and field control, a wide range of speed control is possible.
- The relationship between armature current and torque

$$T_e = K\phi_f i_a$$

- The rated torque

$$T_{er} = K\phi_{fr} i_{ar}$$

- The normalized version

$$T_{en} = \frac{T_e}{T_{er}} = \frac{K\phi_f i_a}{K\phi_{fr} i_{ar}} = \left(\frac{\phi_f}{\phi_{fr}}\right)\left(\frac{i_a}{i_{ar}}\right) = \phi_{fn} i_{an}, \text{ p.u.(per unit)}$$

- Similarly, the air gap power is

  $P_{an} = e_n i_{an}$, p.u.

  where e_n is the normalized induced emf.

- As i_{an} is set to 1 p.u., the normalized air gap power becomes

  $P_{an} = e_n$, p.u.

- The steady-state power output is kept from exceeding its rated design value, which is 1 p.u.

- The air gap power constrains the induced emf and flux field as

$$P_{an} = 1 \text{ p.u.} = e_n i_{an} = \phi_{fn}\omega_{mn} i_{an}$$

- If i_{an} is equal to 1 p.u., then

$$\phi_{fn}\omega_{mn} = 1 \quad \phi_{fn} = 1/\omega_{mn}$$

- Hence, the normalized induced emf is

$$e_n = 1$$

- In the field weaken region, the power output and induced emf are maintained at their rated values by programming the field flux to be inversely proportional to the rotor speed.

3.2.5 Four-quadrant operation

- During a steady speed of ω_m, stoping a machine need to slow down at zero speed first.
- Four-quadrant dc motor drive characteristic

function	Quadrant	Speed	Torque	Power
FM	I	+	+	+
FR	IV	+	-	-
RM	III	-	-	+
RR	II	-	+	-

- Fig. 3.4 shows four-quadrant torque-speed characteristics

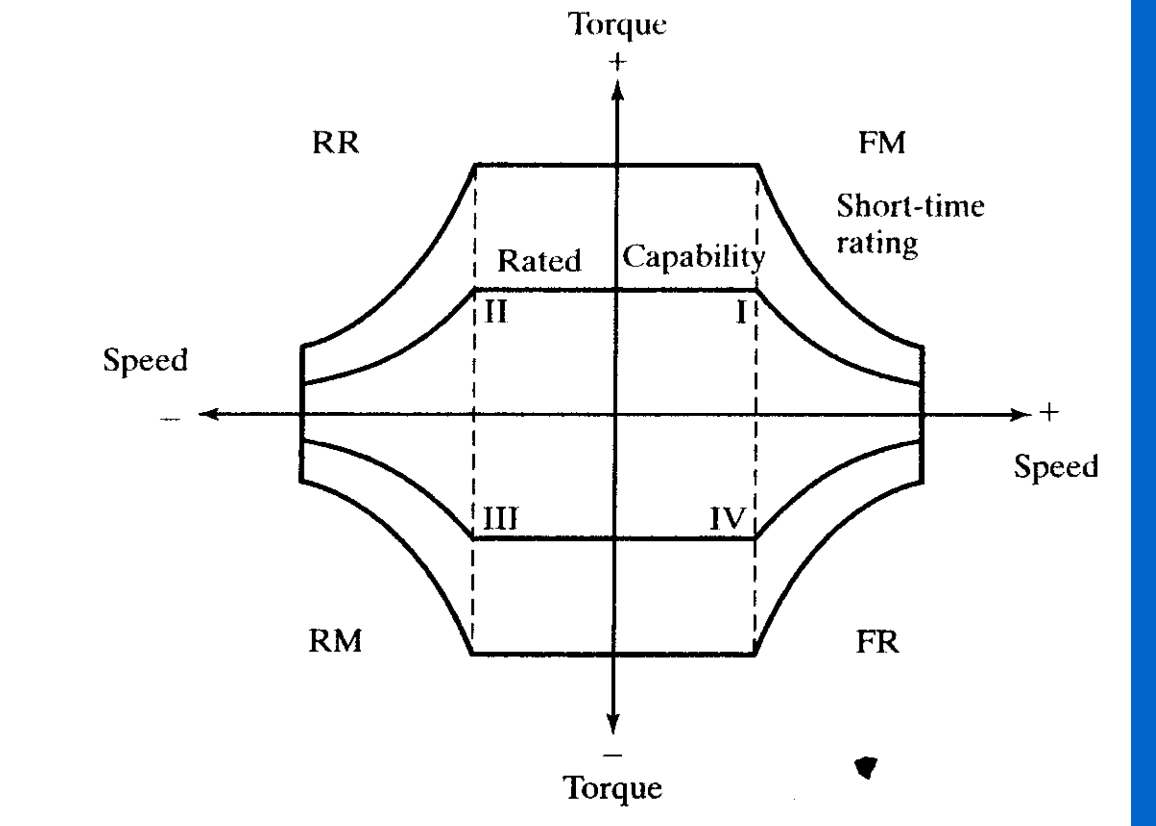


Figure 3.4 Four-quadrant torque–speed characteristics

- Fig. 3.5 illustrates the speed and torque variation.

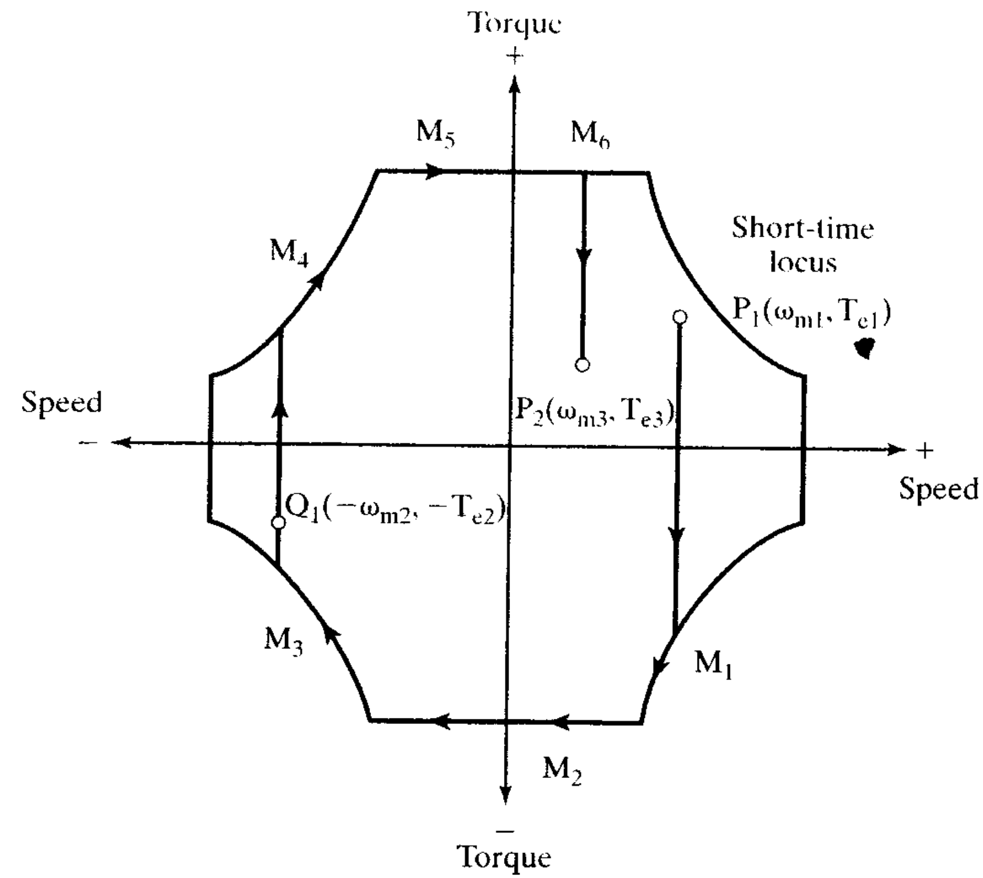


Figure 3.5 Changing the operating points and the use of four quadrants

- Converter requirements: The voltage and current requires four quadrant operation.
- The relationship between armature voltage and armature current

Operation	Speed	Torque	Voltage	Current	Power Output
FM	+	+	+	+	+
FR	+	-	+	-	-
RM	-	-	-	-	+
RR	-	+	-	+	-

- Two basic methods by using static converter

The First method: using phase-controller converter to converter the ac source voltage directly into a variable dc voltage.

The second method: Ac source voltage $\rightarrow$ fixed dc voltage $\rightarrow$ variable dc voltage.

- Thyristor devices:

SCR, Transistors, GTOs, MOSFETs, ect.

3.3 Phase-controlled converter

- Fig 3.6 is a single-phase controlled-bridge converter (positive average value).

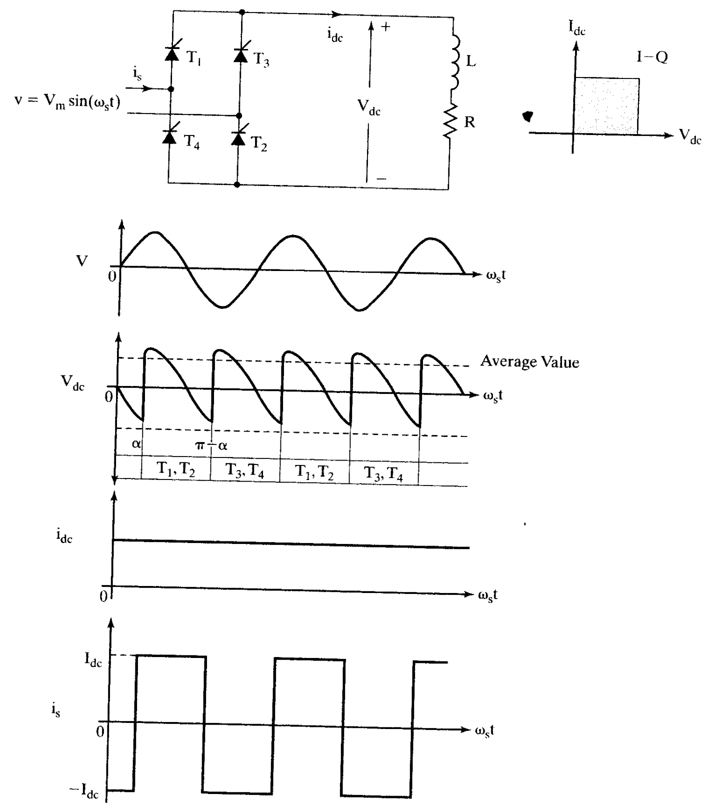


Figure 3.6 A single-phase controlled-bridge converter operating as a rectifier in steady state

- The bridge conduction is delayed in latter beyond positive zero crossing.
- The delay angle is measured from the zero crossing of voltage waveform and is generally termed α.
- Thus, this voltage is quantified as

$$V_{dc} = \frac{1}{\pi}\int_{\alpha}^{\alpha+\pi} V_m \sin(\omega_s t)d(\omega_s t) = \frac{2V_m}{\pi}\cos\alpha$$

- Fig. 3.7 is the controlled-converter operation with negative average voltage ($\alpha > 90^o$).

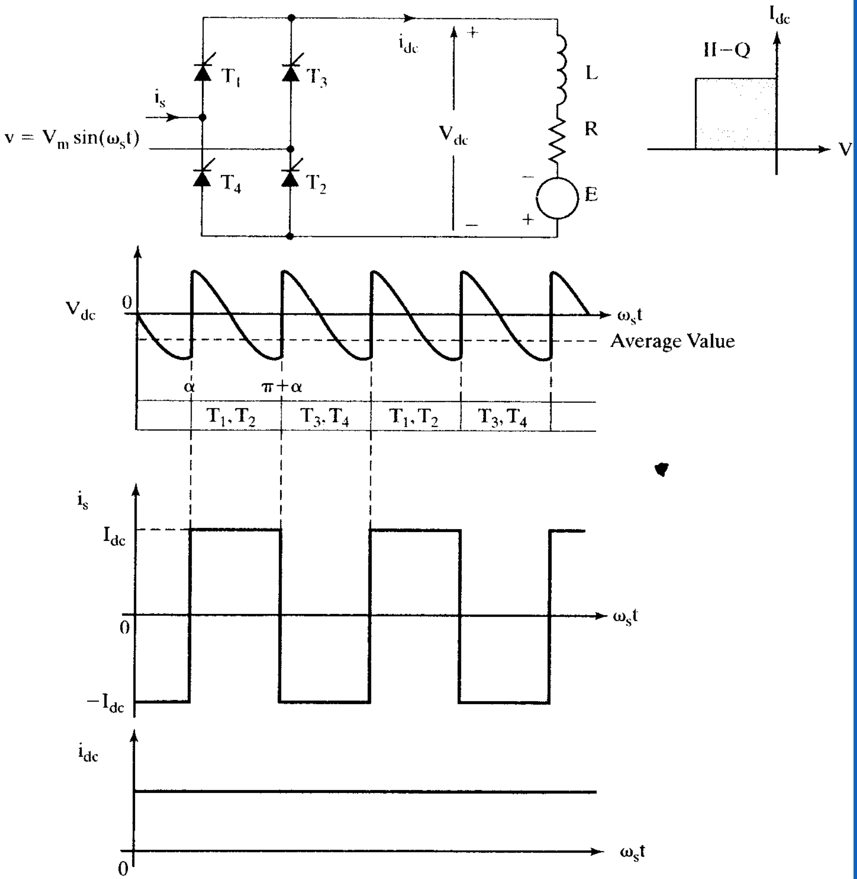


Figure 3.7 Inversion in the steady state of controlled-converter operation

- In the case that the load current is discontinuous, the average output is

$$V_{dc} = \frac{1}{\pi}\int_{\alpha}^{\alpha+\gamma} V_m \sin(\omega_s t)d(\omega_s t) = \frac{V_m}{\pi}[\cos(\alpha) - \cos(\alpha+\gamma)]$$

where γ is the current conduction angle.

- For certain value of γ, the output voltage for discontinuous conduction can be greater than that for continuous conduction.
- For example, let $\alpha+\gamma=\pi$, and $\alpha = 30^\circ$

$$V_{dc}(dis) = \frac{V_m}{\pi}[\cos\alpha - \cos(\alpha+\gamma)] = \frac{V_m}{\pi}1.866$$

$$V_{dc}(con) = \frac{2V_m}{\pi}\cos\alpha = \frac{2V_m}{\pi}\frac{1.732}{2} = \frac{1.732V_m}{\pi}$$

- The source inductance can be introduced to reduce the rate of rise of current in the thyristors.
- If the source inductance is L_{ls}, the voltage lost due to it is

$$V_x = \frac{1}{\pi}\int_{\alpha}^{\alpha+\mu} V_m \sin(\omega_s t)d(\omega_s t) = \frac{V_m}{\pi}[\cos\alpha - \cos(\alpha+\mu)]$$

where , the overlap conduction period is

$$\mu = \cos^{-1}[\cos\alpha - \frac{\pi\omega_s L_{ls} I_{dc}}{V_m}] - \alpha$$

- Fig. 3.10 is a three-phase thyristor-controlled converter.

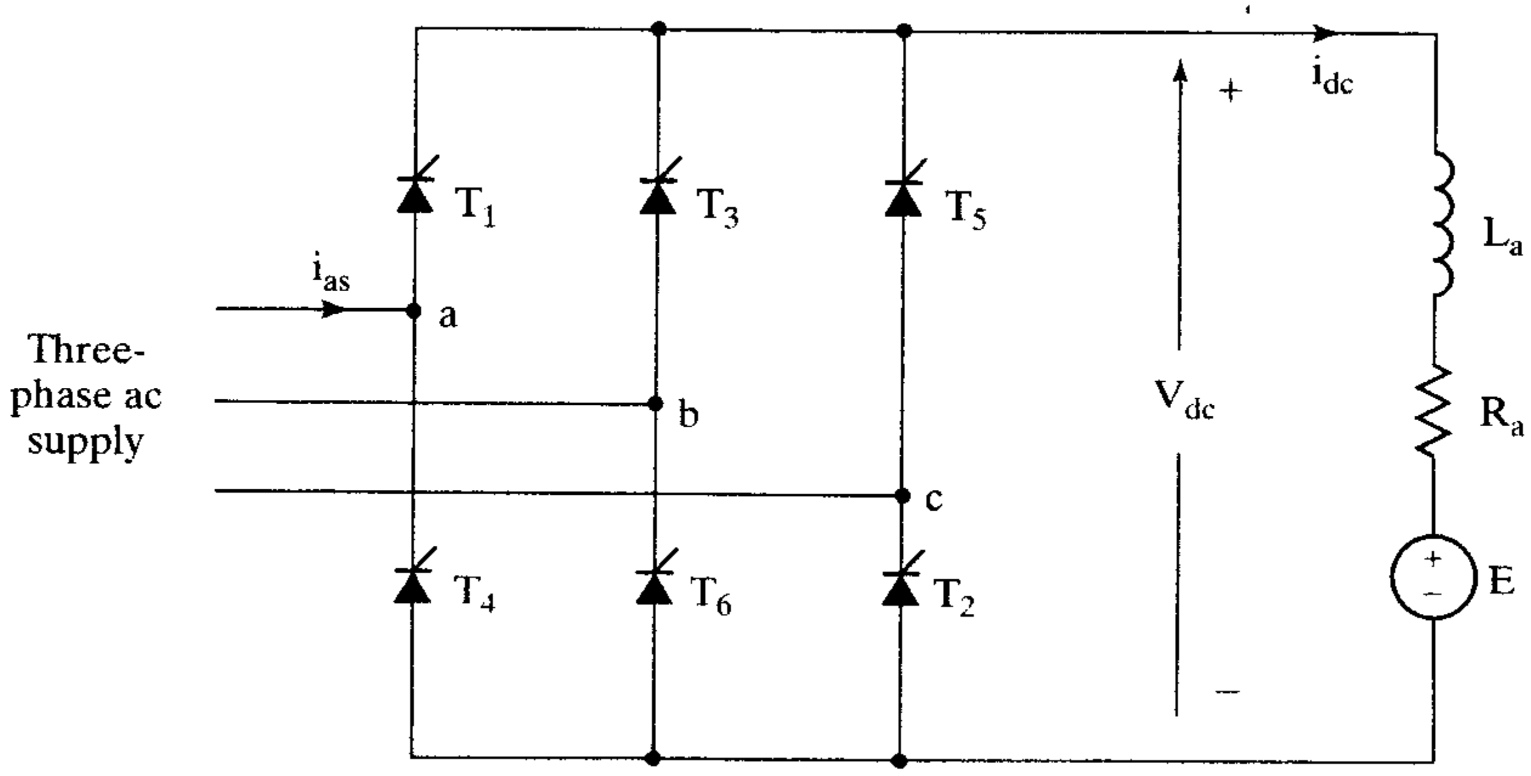


Figure 3.10 Three-phase controlled converter

- The thyristor requires small reactors in series to limit the rate of current rise, and snubbers, which are resistors in series with capacitors across the devices, to limit the rate of voltage rise.
- The transfer characteristic of the three-phase controlled rectifier is derived as

$$V_{dc} = \frac{1}{\pi/3}\int_{\pi/3+\alpha}^{2\pi/3+\alpha} V_m \sin(\omega_s t)d(\omega_s t) = \frac{3}{\pi} V_m \cos\alpha$$

- The characteristic is nonlinear (as shown in Fig. 3.13).

- A control technique to overcome this nonlinear characteristic is that the control input to determine the delay angle is modified to be

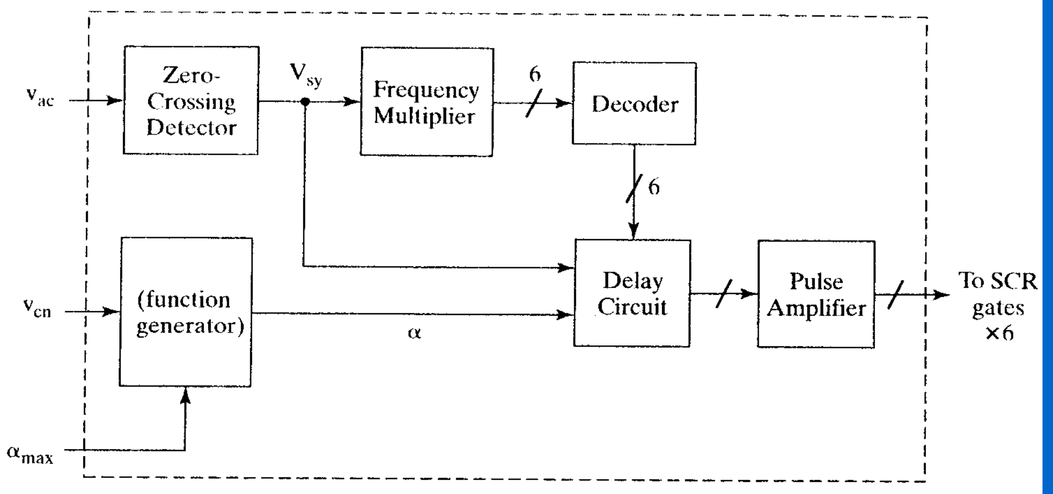
$$\alpha = \cos^{-1}\left(\frac{v_c}{V_{cm}}\right) = \cos^{-1}(v_{cn})$$

where v_c is the control input and V_{cm} is the maximum of the absolute value.

- Then the dc output voltage is

$$V_{dc} = \frac{3}{\pi} V_m \cos\alpha = \frac{3}{\pi} V_m \cos(\cos^{-1} v_{cn}) = [\frac{3}{\pi} V_m] v_{cn} = \left[\frac{3}{\pi}\frac{V_m}{V_{cn}}\right] v_c = K_r v_c$$

- Fig. 3.14 is a schematic of a generic implementation.
- The maximum delay angle is usually set in the range from 150 to 155 degree.



Note : v_{ac} Control level voltage signal between phases *a* and *c*
v_{cn} Normalized control voltage
V_{sy} Synchronizing signal
α_{max} Maximum limit for the delay angle

Figure 3.14 Controller schematic for the three-phase converter

Control modeling

- The gain of the linearized controller-based converter is

$$K_r = \frac{1.35V}{V_{cm}},$$

- The converter is a sampled-data system. The sampling interval gives an indication of its time delay.
- The delay may be treated as one half of this interval

$$T_r = \frac{60/2}{360} \times (\text{time period of one cycle}) = \frac{1}{12} \times \frac{1}{f_s}$$

- The converter is then modeled with

$$G_r(s) = K_r e^{-T_r s}$$

- The above equation can also be approximated by as

$$G_r(s) = \frac{K_r}{(1 + sT_r)}$$

- For the case that the transfer characteristic is nonlinear, the gain of the converter is obtained as a small-signal gain by

$$K_r = \frac{\delta V_{dc}}{\delta \alpha} = \frac{\delta}{\delta \alpha}\{1.35V\cos\alpha\} = -1.35\sin\alpha$$

- Fig. 3.15 is current source converter.

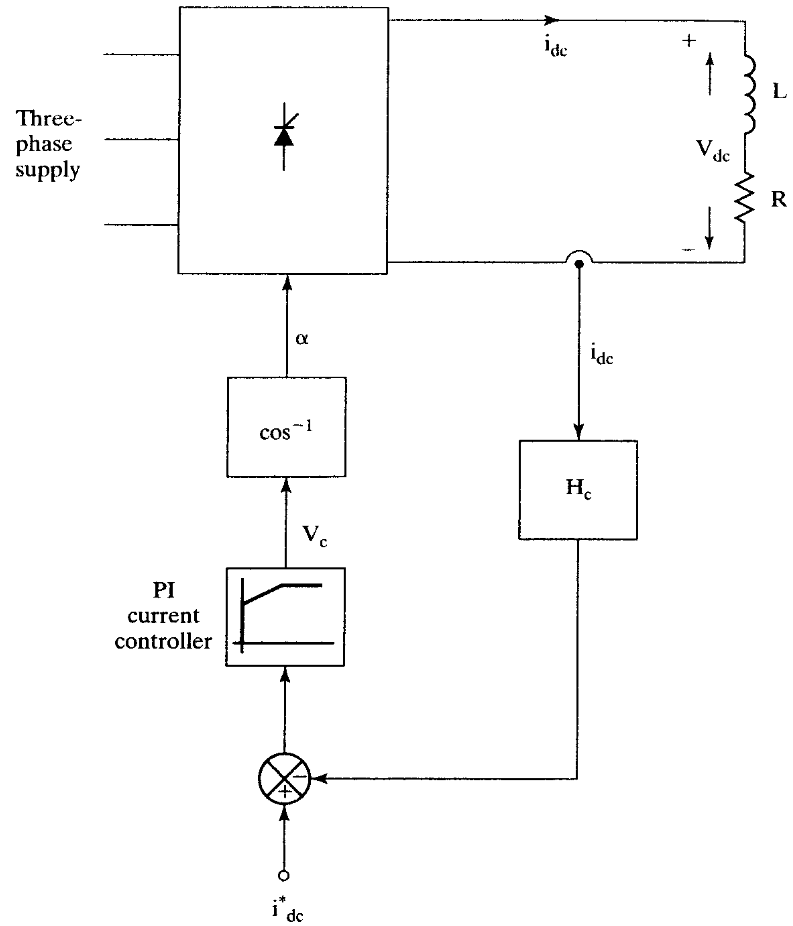


Figure 3.15 Current source with three-phase controlled converter

• Fig. 3.16 current source operation.

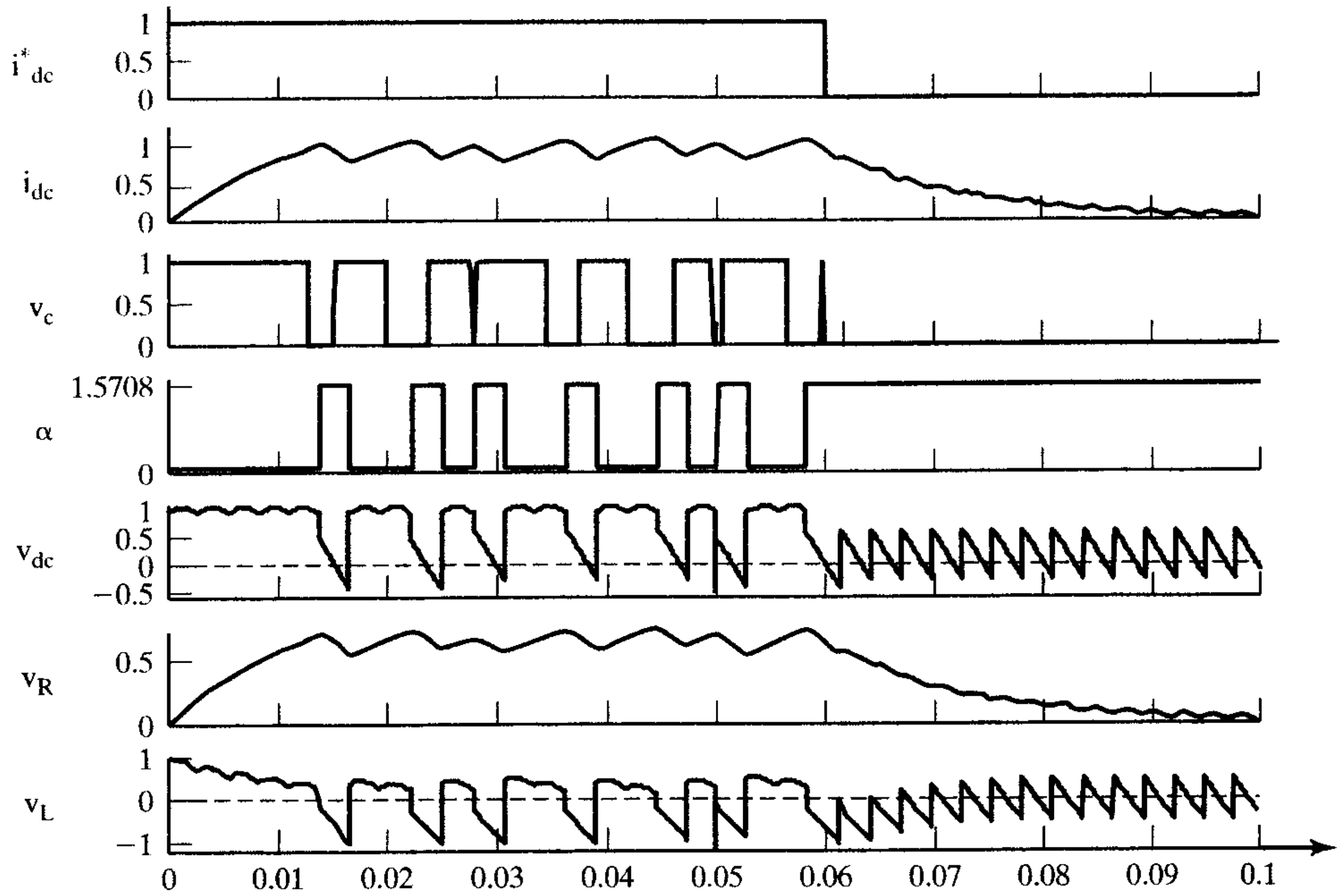


Figure 3.16 Current source operation with R-L load in normalized units

- Fig. 3.17 is a half-controlled converter.

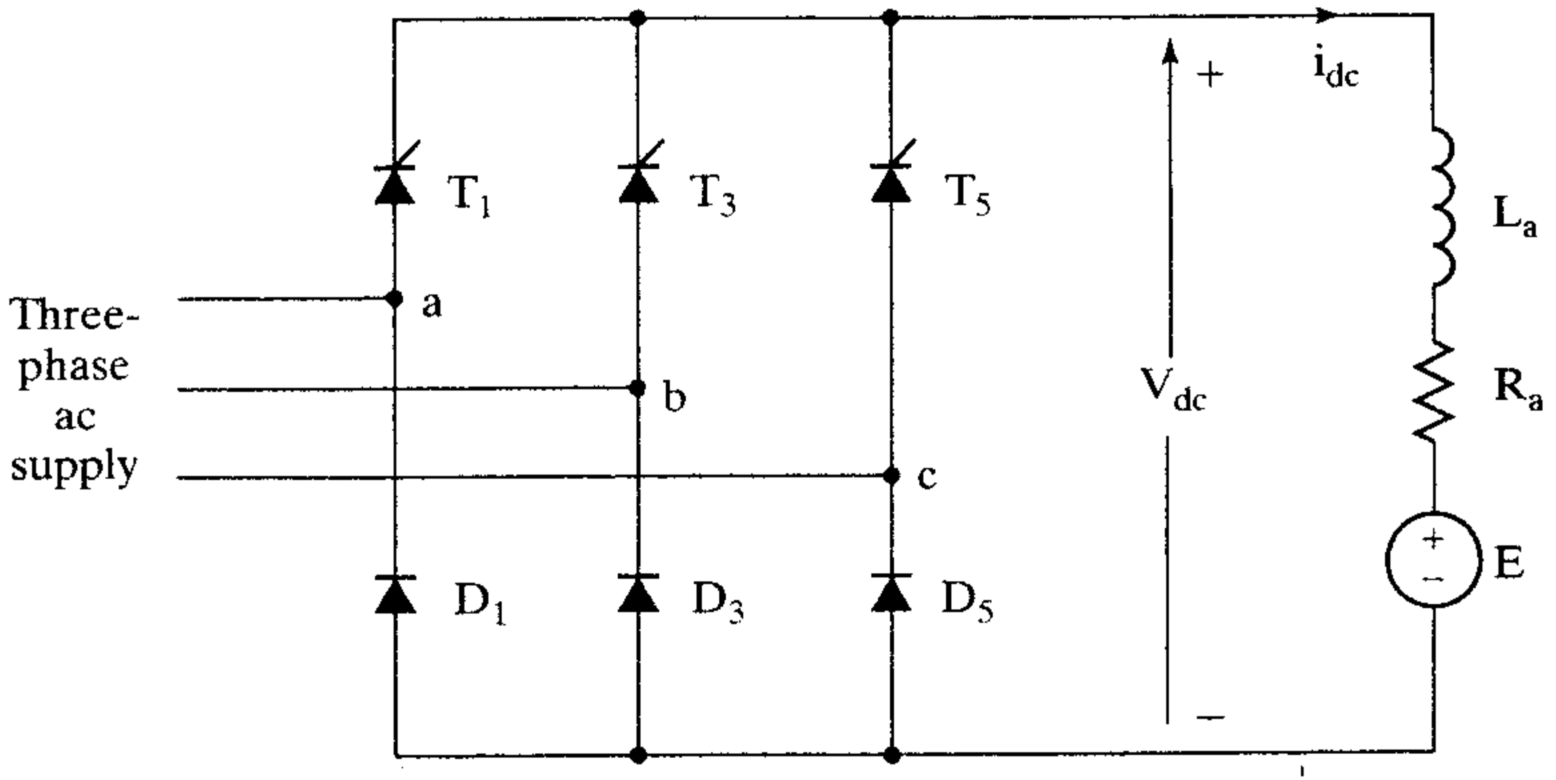


Figure 3.17 Half-controlled three-phase converter

- Fig. 3.18 is the converter with freewheeling diode.

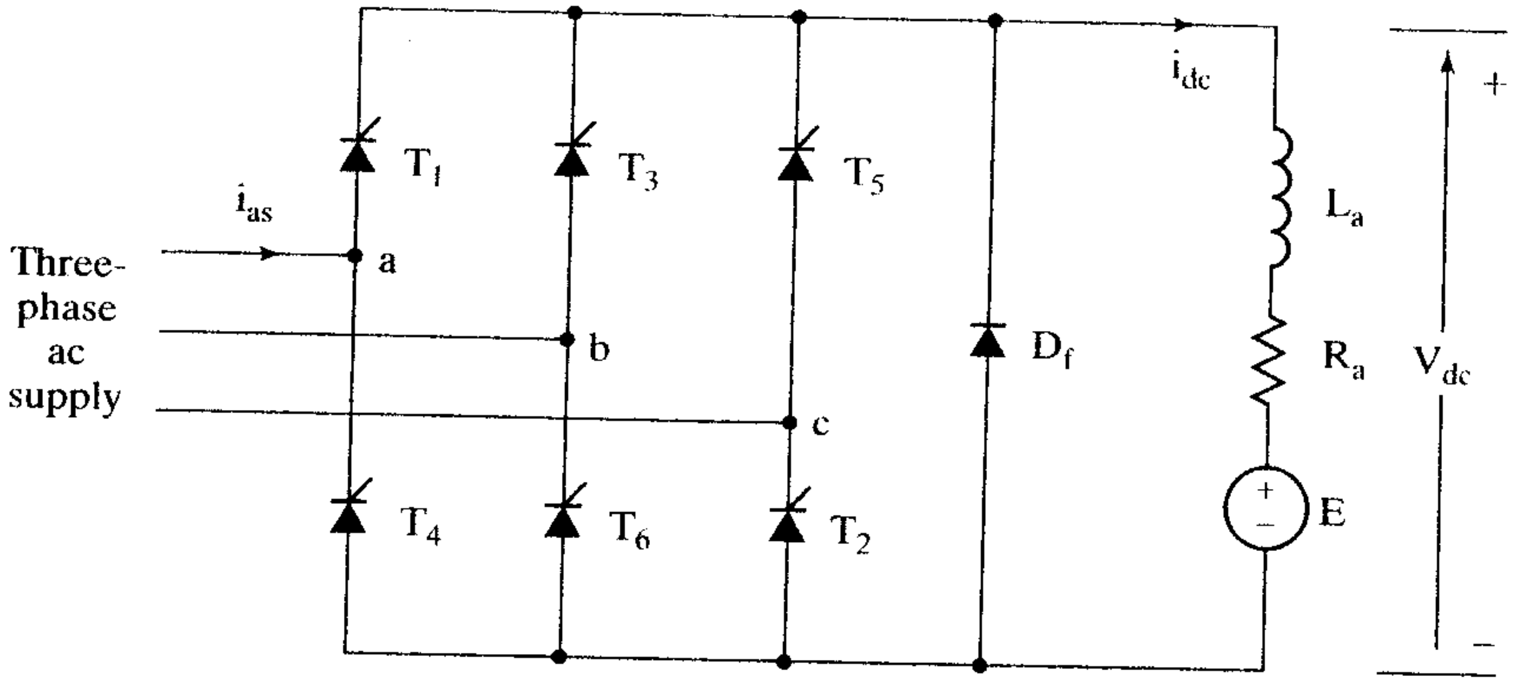


Figure 3.18 Three-phase controlled converter with freewheeling

- Fig. 3.20 shows the converter configuration for a four-quadrant dc motor drive.

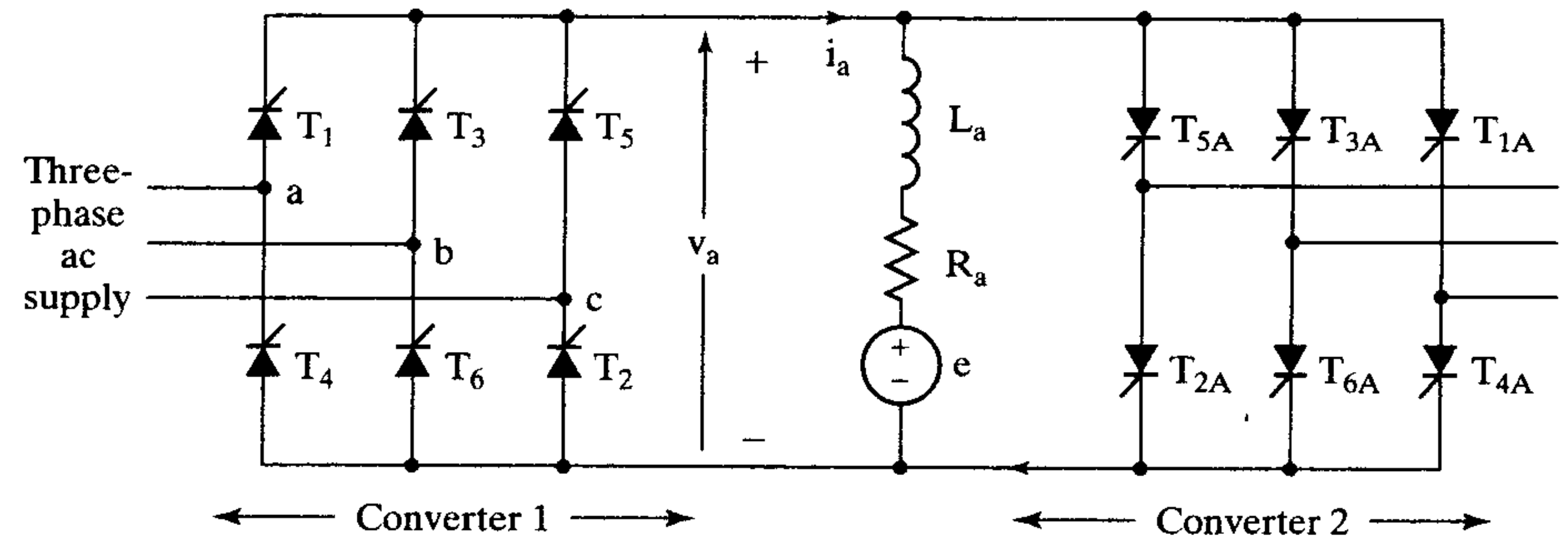


Figure 3.20 Dual three-phase thyristor converter for four-quadrant drive

3.4 Steady-state analysis ...

- Average values: The steady-state performance is developed by assuming that the average values only are considered.
- The armature voltage equation

$$v_a = R_a i_a + e$$

- Termed by average values

$$V_a = R_a I_a + K\Phi_f \omega_{mav}$$

- Average electromagnetic torque is

$$T_{av} = K\Phi_f I_a = K\Phi_f \{\frac{V_a - K\Phi_f \omega_{mav}}{R_a}\} = K\Phi_f \{\frac{1.35V \cos\alpha - K\Phi_f \omega_{mav}}{R_a}\}$$

- The normalized electromagnetic torque

$$T_{en} = \frac{T_{av}}{T_{er}} = \frac{T_{av}}{K\Phi_{fr} I_{ar}} = \frac{K\Phi_f \{1.35V\cos\alpha - K\Phi_f \omega_{mav}\}}{K\Phi_{fr} I_{ar} R_a} = \frac{[1.35V\cos\alpha - K\Phi_f \omega_{mav}]}{I_{ar} R_a} \Phi_{fn}$$

$$T_{en} = \frac{[1.35V_{en}\cos\alpha - \Phi_{fn}\omega_{mn}]}{R_{an}} \Phi_{fn}, \ \text{p.u.}$$

- Positive and motoring torque is produced when

$$\cos\alpha > \frac{\Phi_{fn}\omega_{mn}}{1.35V_n}$$

- Steady-state solution including harmonics accurately predict its electromagnetic torque
- The speed of the machine and the field current are assumed to be constant. Then

$$R_a i_a + L_a \frac{di_a}{dt} + K_b \omega_m = v_a$$

where $v_a = V_m \sin(\omega_s t + \pi/3 + \alpha)$, $0 < \omega_s t < \pi/3$

- The induced emf is a constant under the assumption of constant speed, hence its solution is

$$i_a(t) = \left(\frac{V_m}{|Z_a|}\right)\{\sin(\omega_s t + \pi/3 + \alpha - \beta) - \sin(\pi/3 + \alpha - \beta)e^{-t/T_a}\} - \left(\frac{E}{R_a}\right)(1 - e^{-t/T_a}) + i_{ai}e^{-t/T_a}$$

where $\omega_s = 2\pi f_s$, $\beta = \tan^{-1}(\omega_s L_a/R_a)$ = machine impedance angle, $T_a = L_a/R_a$ = armature time constant, i_{ai} = initial value of current at time t = 0, $Z_a = R_a + j\omega_s L_a$ = motor electrical impedance.

- Critical triggering angle α_c: when the armature current is barely continuous (iai = 0).

$$\alpha_c = \beta + \cos^{-1}\{\frac{(E/V_m)}{c_1} \cdot \frac{1}{\cos\beta} \cdot (1 - e^{-(\pi/3\tan\beta)})\} - \frac{\pi}{3} + \theta_1$$

- When the current becomes discontinuous, the voltage across the machine the is the induced emf itself.

- The steady state is

$$R_a i_a + L_a \frac{di_a}{dt} + E = V_a, \quad 0 < \omega_s t < \omega_s t_x$$

$$i_a = 0, \quad \omega_s t_x < \omega_s t < \pi/3$$

$$V_a = V_m \sin(\omega_s t + \pi/3 + \alpha), \quad 0 < \omega_s t < \omega_s t_x$$

$$= E, \quad \omega_s t_x < \omega_s t < \pi/3$$

• Fig 3.24

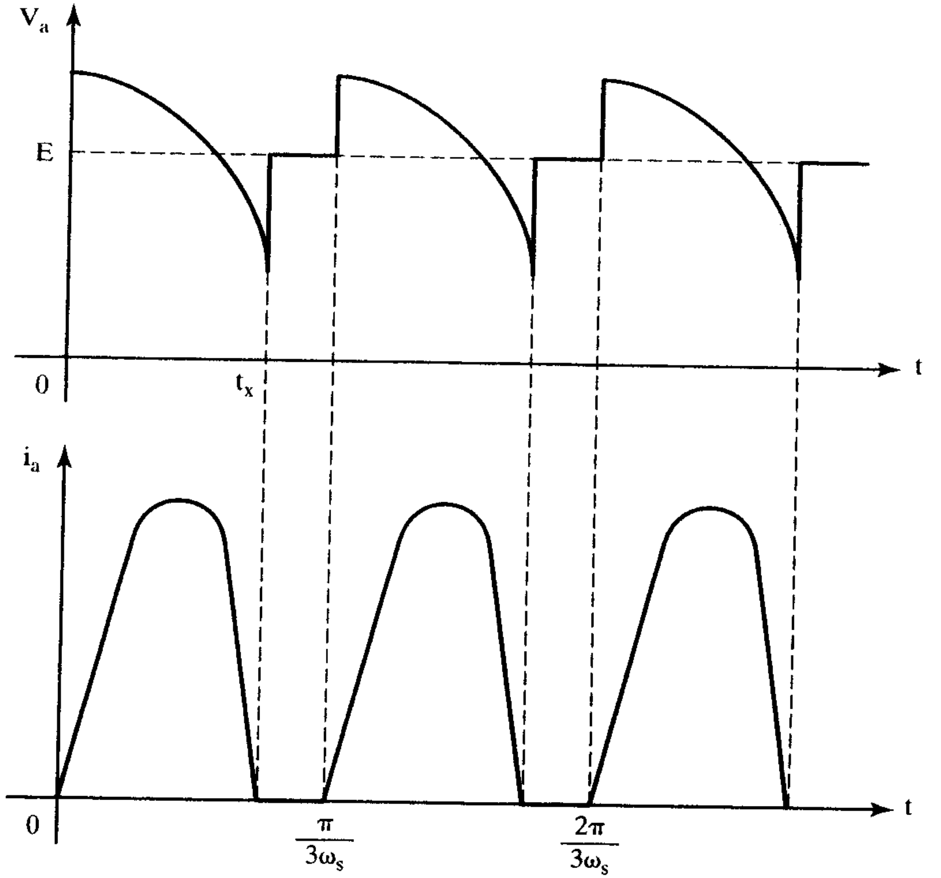


Figure 3.24 Discontinuous current conduction waveforms

3.5 Two-Quadrant three-phase converter-controlled DC motor drive

- Fig. 3.26 is speed-controlled two-quadrant dc motor drive

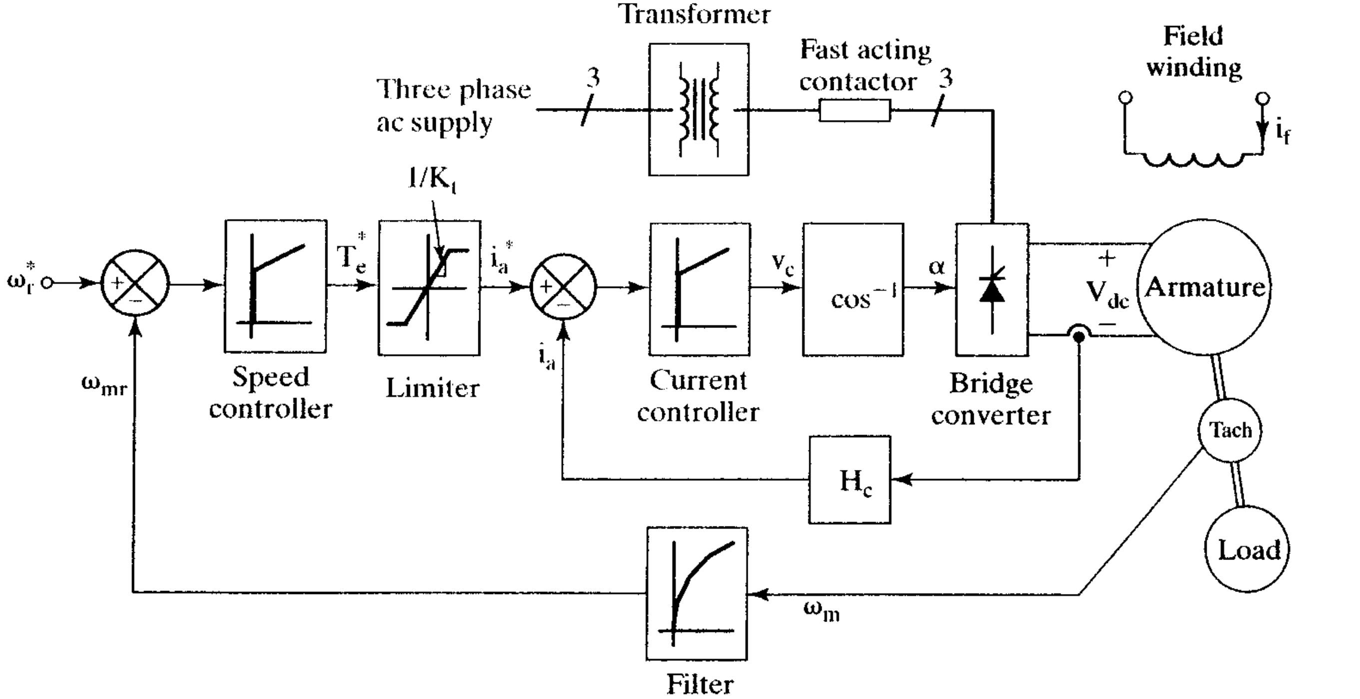


Figure 3.26 Speed-controlled two-quadrant dc motor drive

3.6 Transfer functions of the subsystems

- Fig. 3.27 is DC motor and current-control loop.

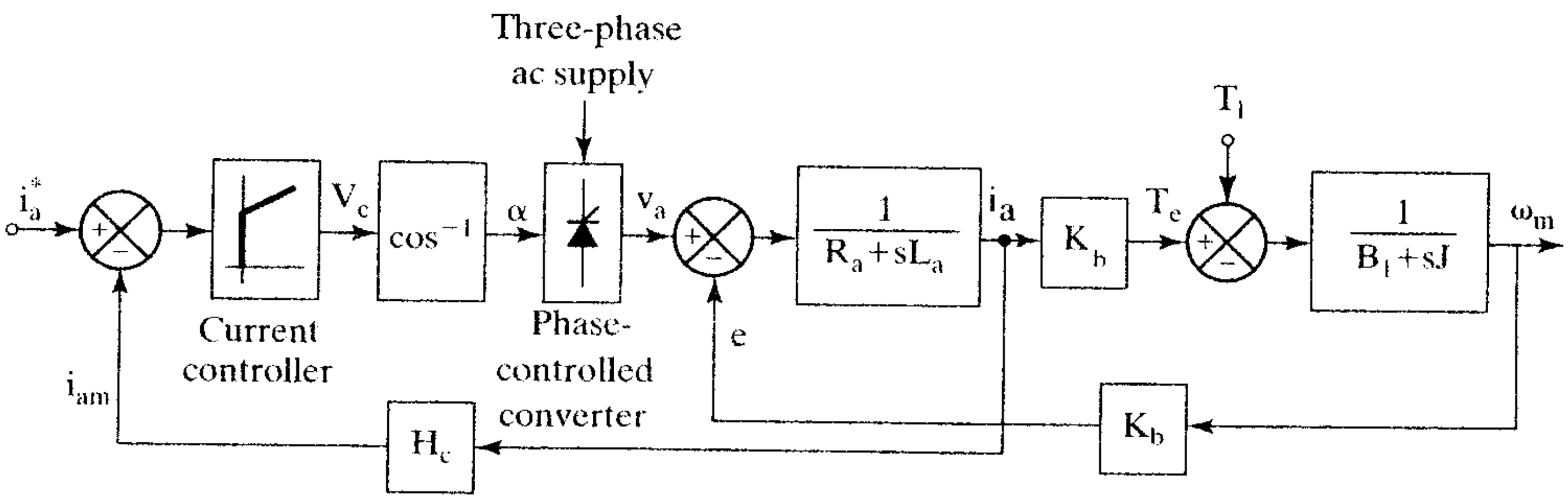


Figure 3.27 DC motor and current-control loop

- Fig. 3.28 is step-by-step derivation of a dc machine transfer function.

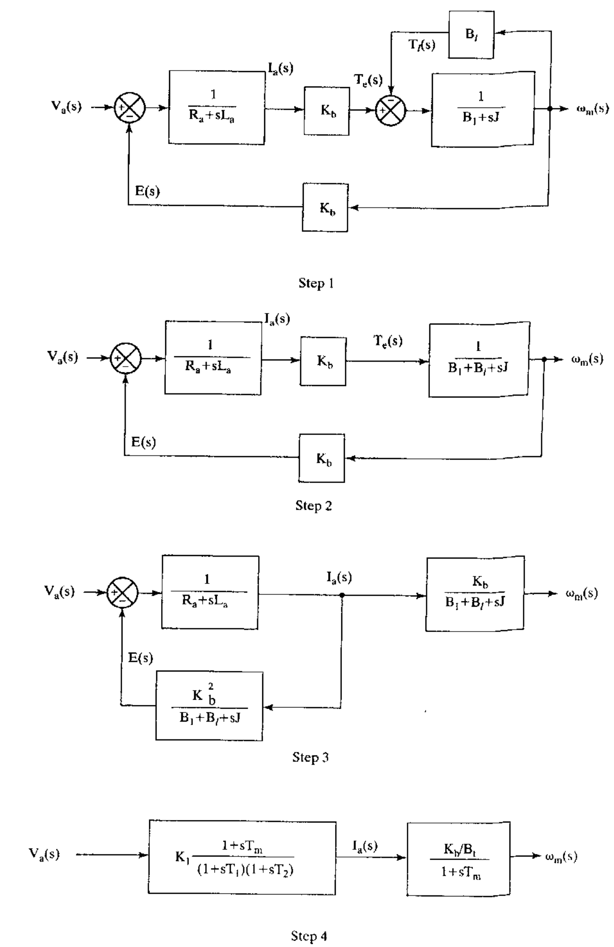


Figure 3.28 Step-by-step derivation of a dc machine transfer function

- Converter (after linearization)

$$G_r = \frac{V_a(s)}{V_c(s)} = \frac{K_r}{1+sT_r}$$

- The current and speed controllers of proportional-integral type are

$$G_c(s) = \frac{K_c(1+sT_c)}{sT_c}$$

$$G_s(s) = \frac{K_s(1+sT_s)}{sT_s}$$

- The gain of current feedback is H_c.
- The transfer function of the speed feedback filter is $G_\omega(s) = \frac{K\omega}{1+sT_\omega}$

3.7 Design of controllers

- Fig 3.29 is the block diagram of the motor drive

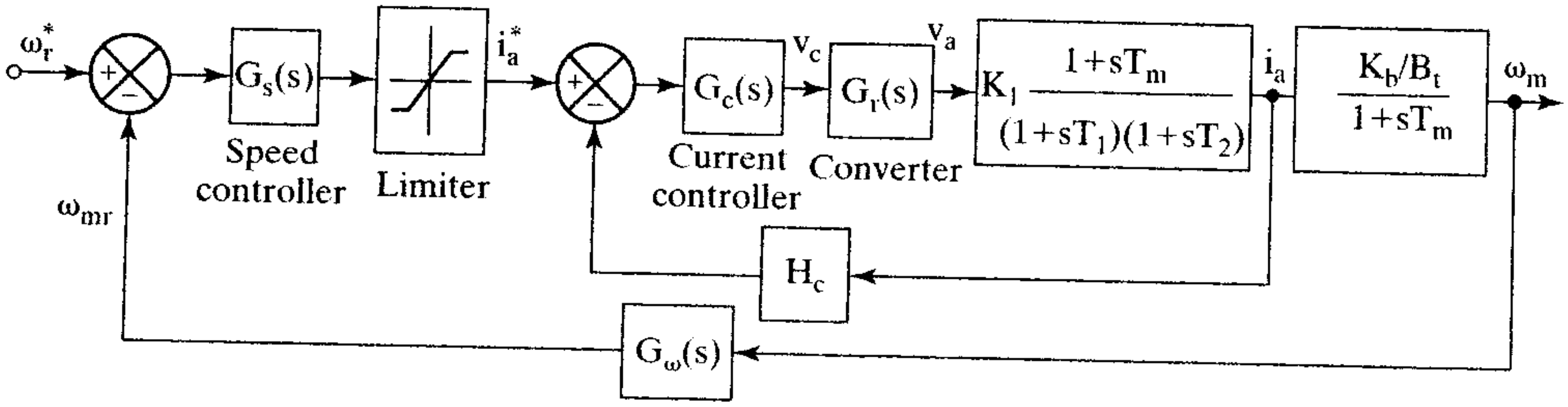


Figure 3.29 Block diagram of the motor drive

- Fig 3.30 is current-control loop.

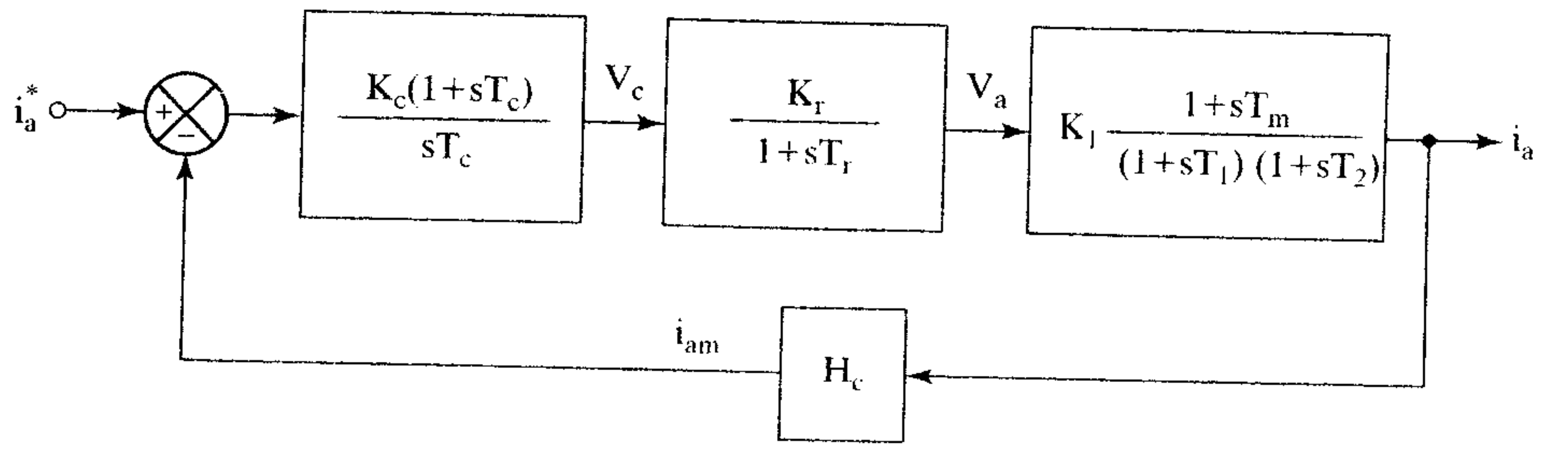


Figure 3.30 Current-control loop

- Speed controller: Fig. 3.32 is the representation of the outer speed loop in the dc motor drive.

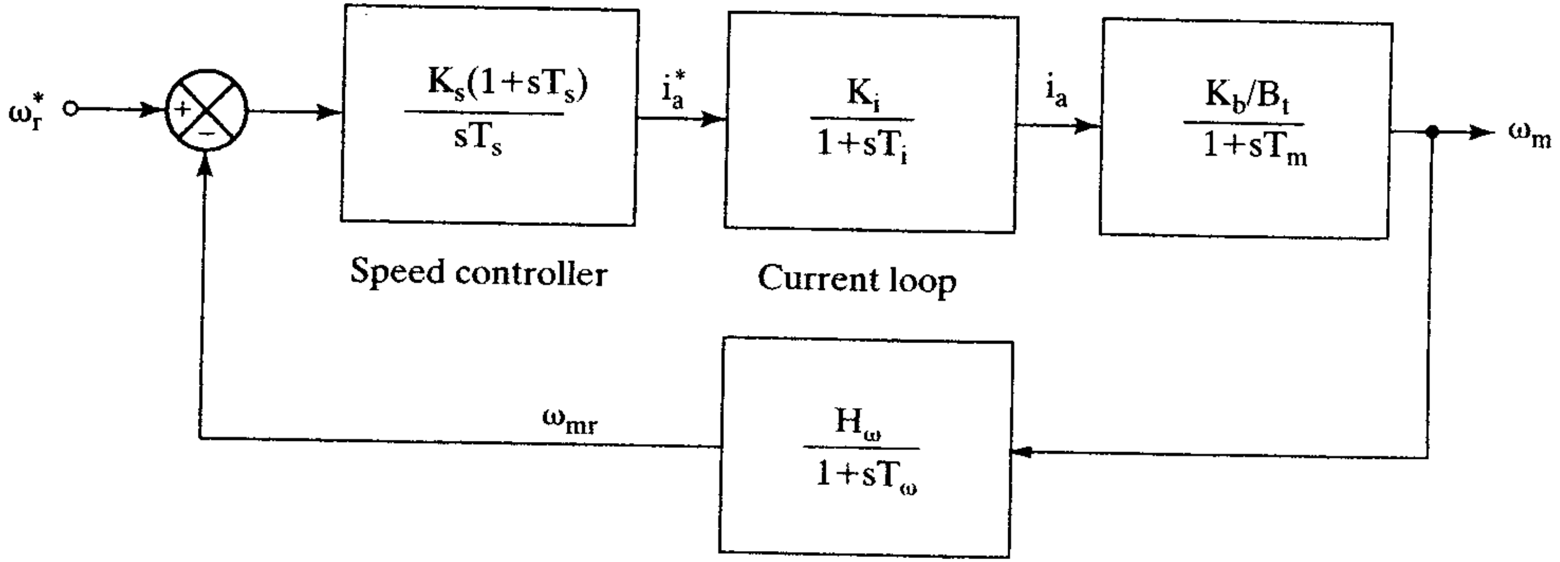


Figure 3.32 Representation of the outer speed loop in the dc motor drive

- The closed-loop transfer function of the speed to its command is

$$\frac{\omega_m(s)}{\omega_r^*(s)} = \frac{1}{H_\omega}\left[\frac{1+4T_4 s}{1+4T_4 s+8T_4^2 s^2+8T_4^3 s^3}\right]$$

3.8 Two-quadrant DC motor drive with field weakening

- Fig. 3.38 is its schematic.

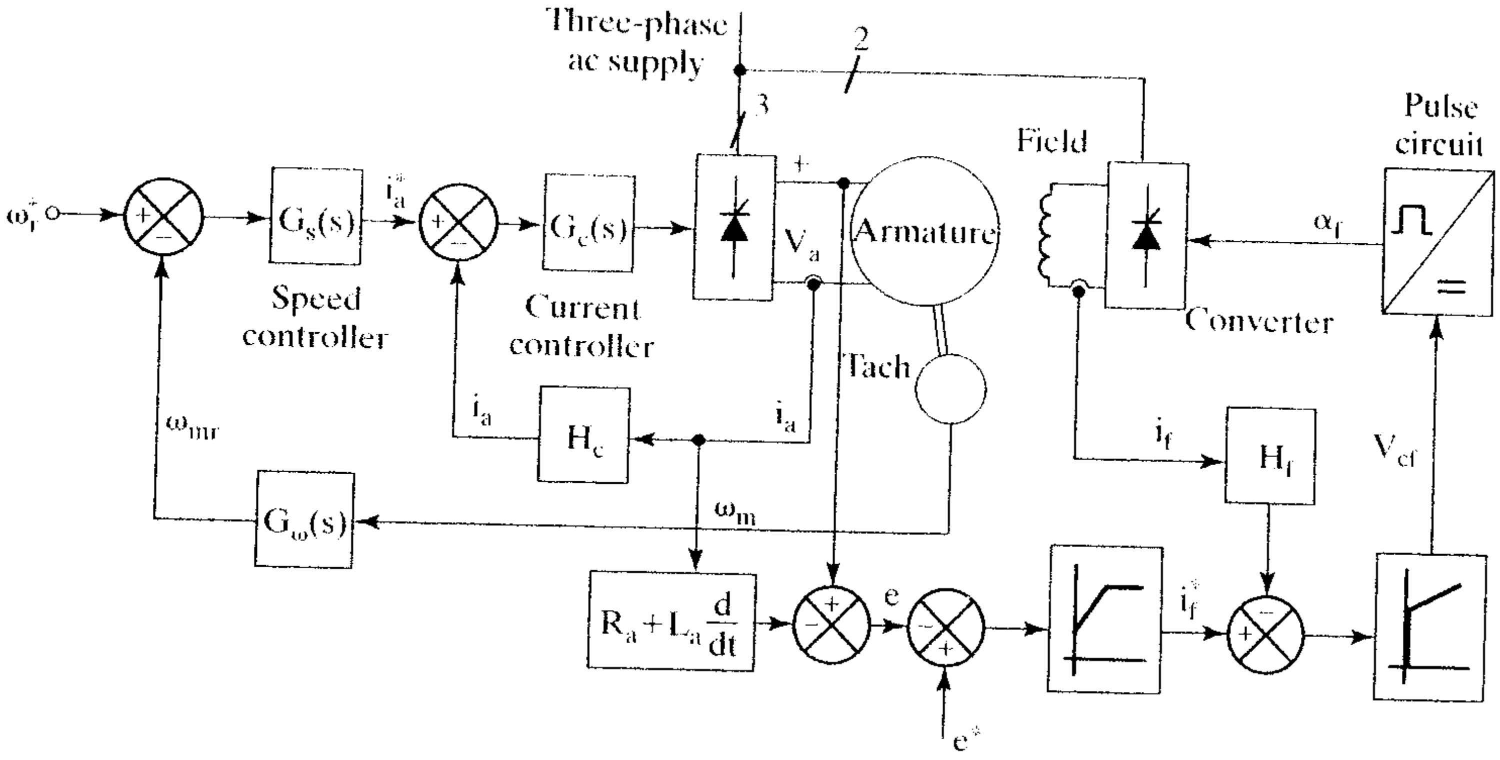
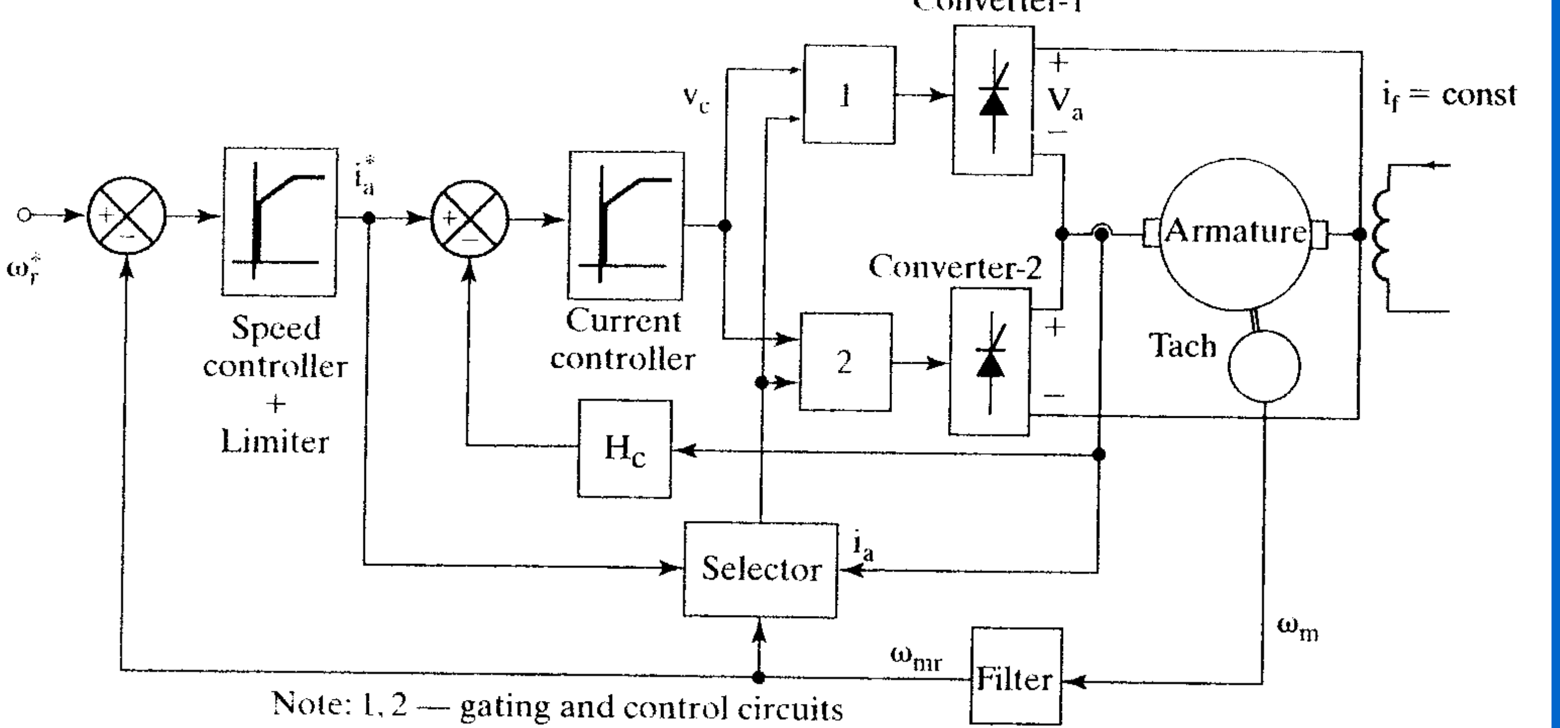


Figure 3.38 A two-quadrant dc motor drive with field weakening

3.9 Four-quadrant Dc motor drive

- Fig. 3.39 is its schematic.



Note: 1, 2 — gating and control circuits

Figure 3.39 A four-quadrant dc motor drive