2 Modeling of DC machine

- 2.1 Theorem of operation
- Maximum torque is produced when two fluxes are in quadrature.

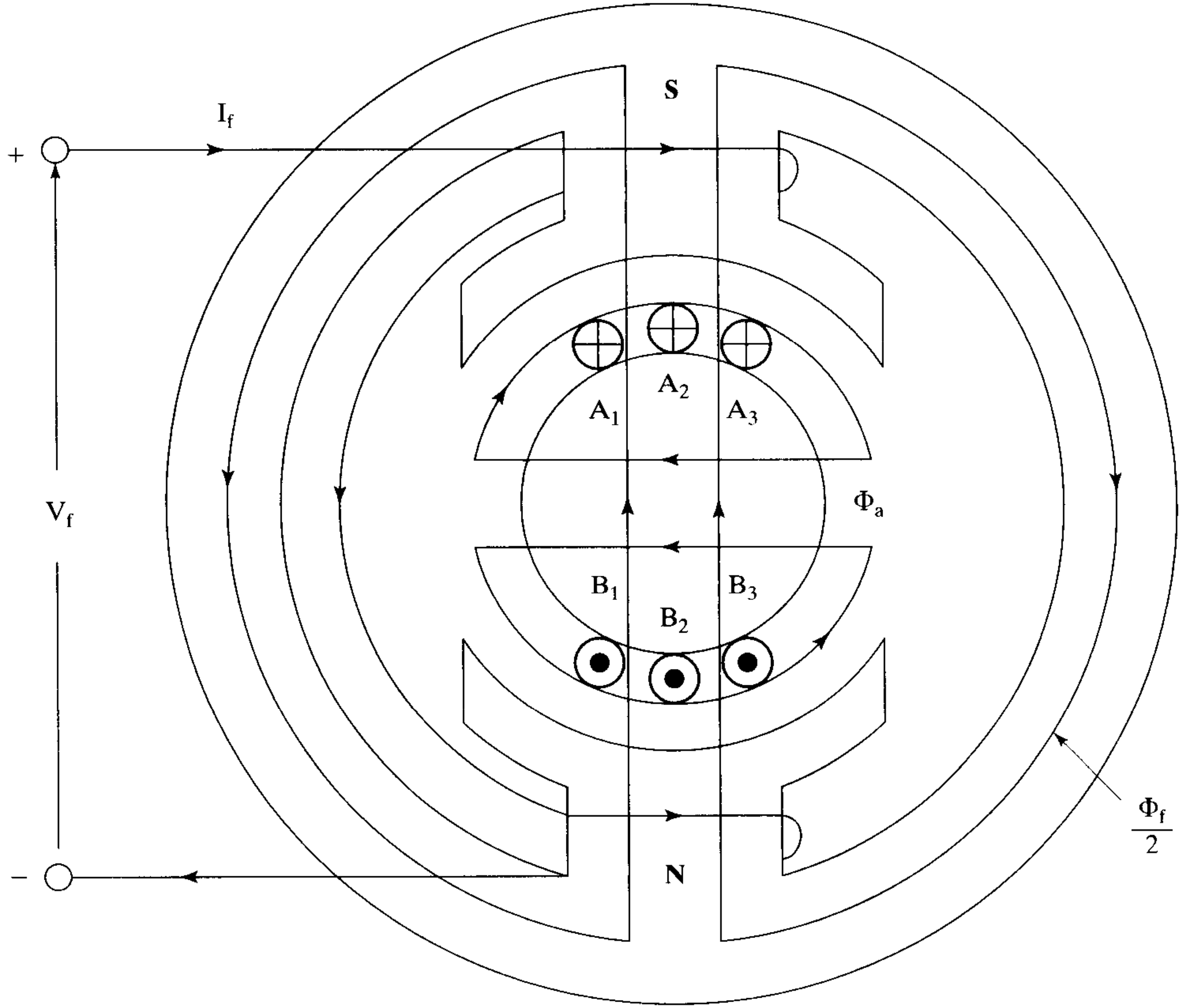

Figure 2.1 Schematic representation of a dc machine

2.2 Induced EMF

- From Faraday's law, the induced emf is

$$e = Z\frac{d\phi_f}{dt} = Z\frac{\phi_f}{t}$$

where t is the time taken by the conductors to cut ϕ_f flux lines. Therefore,

$$t = \frac{1}{2 \times \text{frequency}} = \frac{1}{2(\frac{p}{2})(\frac{n_r}{60})}$$

- Thus,(P: poles; Z: armature conductors; ϕ_f: a flux per pole; n_r: rotation speed)

$$e = \frac{Z\phi_f P n_r}{60}$$

- If the armature conductors are divided into ‘a’ parallel paths, then

$$e = \frac{Z\phi_f P n_r}{60}$$

wave winding: a = 2; lap winding: a = P.

- The usual expression

$e = K\phi_f\omega_m$

where $\omega_m = 2\pi n_r/60$ rad/sec and $K = (P/a)Z(1/2\pi)$

- If the field flux is constant, then emf is

$e = K_b\omega_m$

2.3 Equivalent circuit and electromagnetic torque

- The terminal relationship is

$$v = e + R_a i_a + L_a \frac{di_a}{dt}$$

- In steady state, the armature current is constant and hence

$$v = e + R_a i_a$$

- The power balance

$$v i_a = e i_a + R_a i_a^2$$

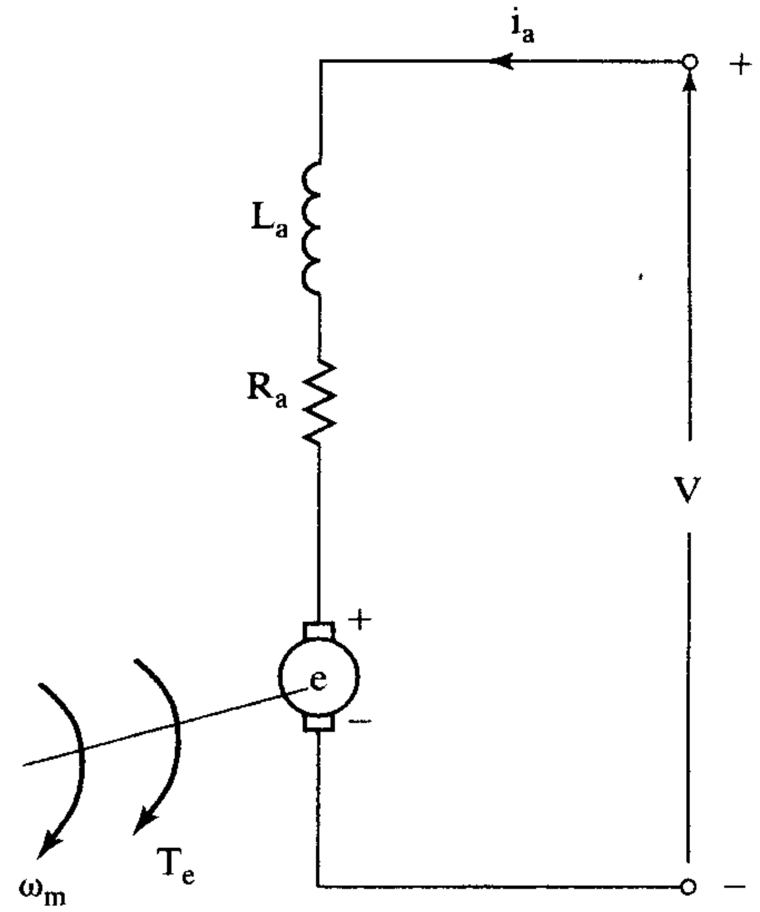

Figure 2.2 Equivalent circuit of a dc motor armature

- The power of mechanical form, P_a called the air gap power, is expressed in term of the electromagnetic torque and speed as

$$P_a = \omega_m T_e = e i_a$$

- Hence,

$$T_e = \frac{e i_a}{\omega_m} = \frac{K_b \omega_m i_a}{\omega_m} = K_b i_a$$

2.4 Electromechanical modeling

- The acceleration torque, Ta, drives the load and is given by

$$J\frac{d\omega_m}{dt} + B_1\omega_m = T_e - T_1 = T_a$$

where J: a moment of inertia (kg-m^2/sec^2)

B_1: a viscous friction coefficient N·m/(rad/sec)

T_1: the load torque

2.5 State-space modeling

- The dynamic equations in state-space form

$$\begin{bmatrix} pi_a \\ p\omega_m \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{K_b}{L_a} \\ \frac{K_b}{J} & -\frac{B_1}{j} \end{bmatrix} \begin{bmatrix} i_a \\ \omega_m \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} & 0 \\ 0 & -\frac{1}{J} \end{bmatrix} \begin{bmatrix} V \\ T_1 \end{bmatrix}$$

- The roots of the system are

$$\lambda_1, \lambda_2 = \frac{-(\frac{R_a}{L_a} + \frac{B_1}{J}) \pm \sqrt{(\frac{R_a}{L_a} + \frac{B_1}{J})^2 - 4(\frac{R_a B_1}{JL_a} + \frac{K_b^2}{JL_a})}}{2}$$

2.6 Block diagrams and transfer functions

- From (2.13) and (2.19), we get

$$I_a(s) = \frac{V(s) - K_b \omega_m(s)}{R_a + sL_a}$$

$$\omega_m = \frac{K_b I_a(s) - T_l(s)}{(B_1 + sJ)}$$

- The transfer functions

$$G_{\omega V}(s) = \frac{\omega_m(s)}{V(s)} = \frac{K_b}{s^2(JL_a) + a(B_1 L_a + JR_a) + (B_1 R_a + K_b^2)}$$

$$G_{\omega l}(s) = \frac{\omega_m(s)}{T_s(s)} = \frac{-(R_a + sL_a)}{s^2(JL_a) + s(B_1 L_a + JR_a) + (B_1 R_a + K_b^2)}$$

- The speed response due to the simultaneous voltage input and load torque disturbance is $\omega_m(s) = G_{\omega V}(s)V(s) + G_{\omega l}(s)T_1(s)$

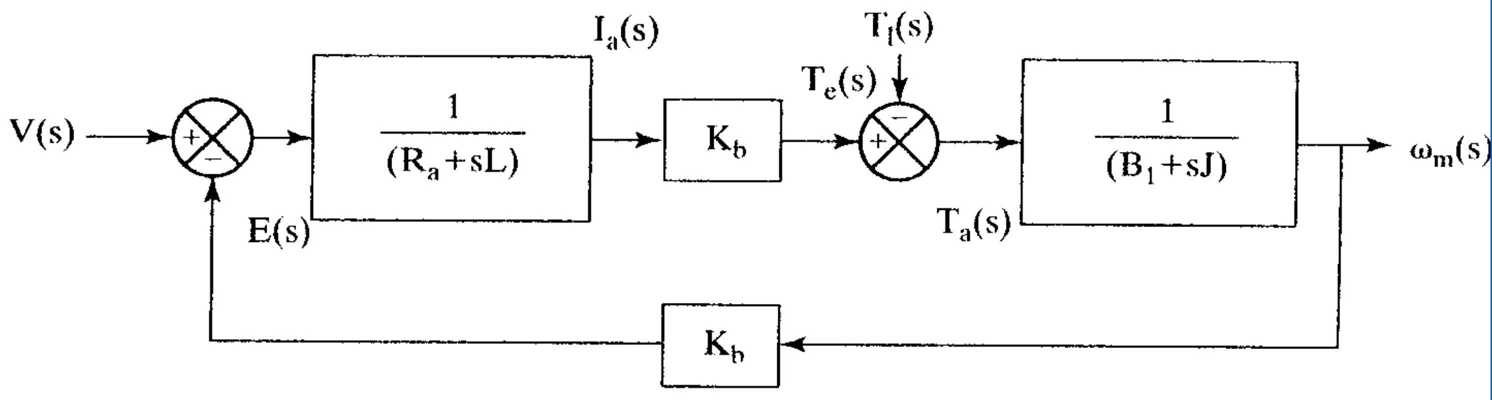

Figure 2.3 Block diagram of the dc motor

2.7 Field excitation

- Separately excited dc machine

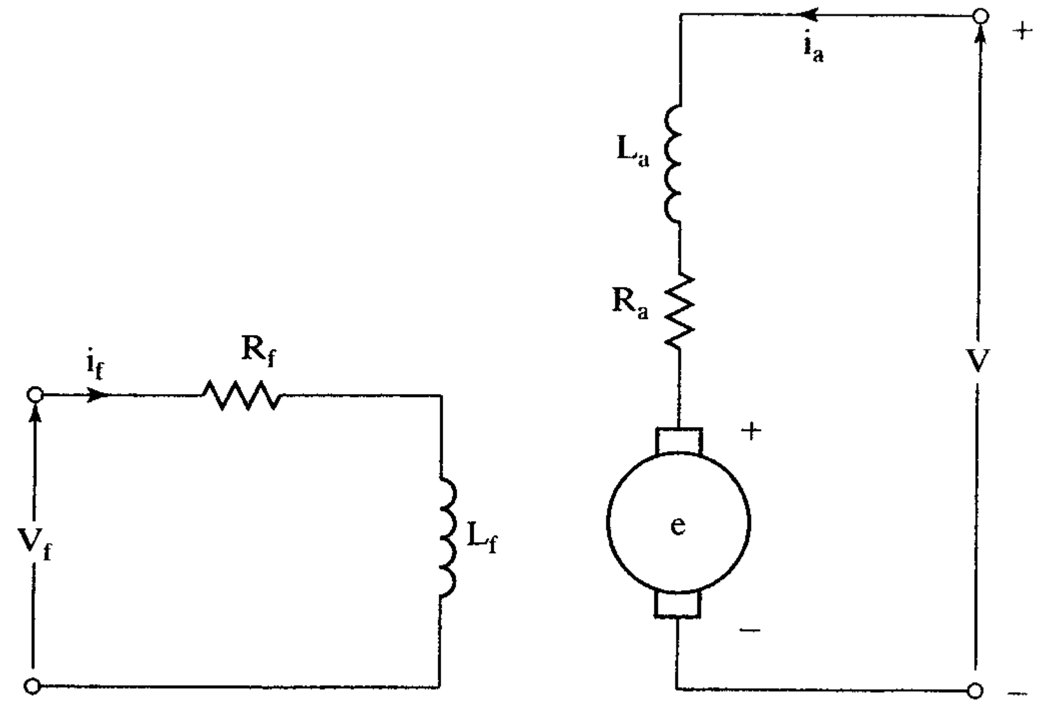

Figure 2.4 Separately-excited dc machine

- Shunt-excited dc machine

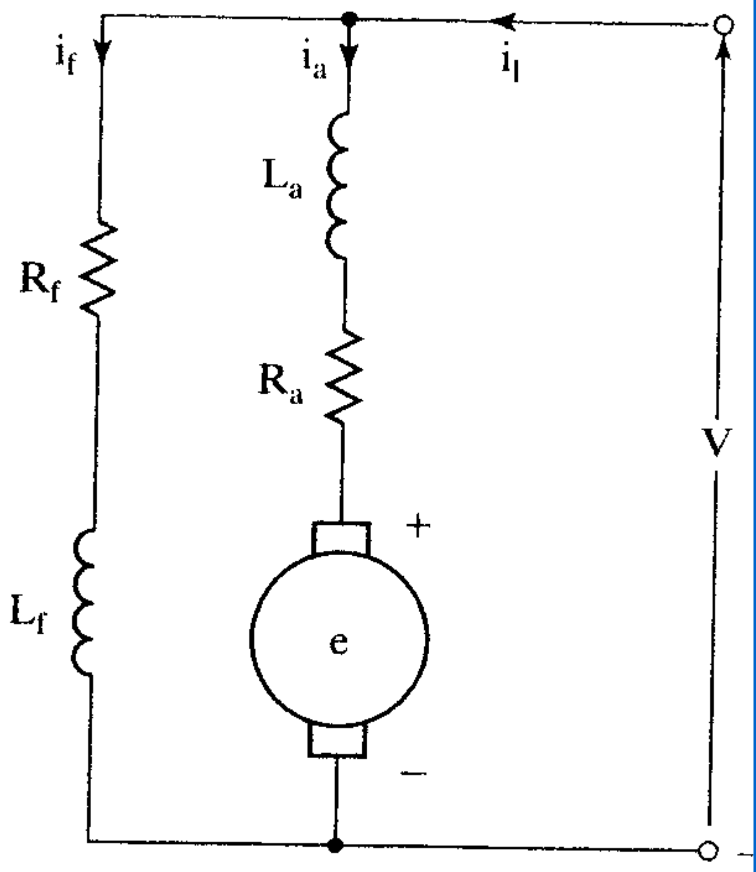

Figure 2.5 dc shunt machine

• Series-excited dc machine

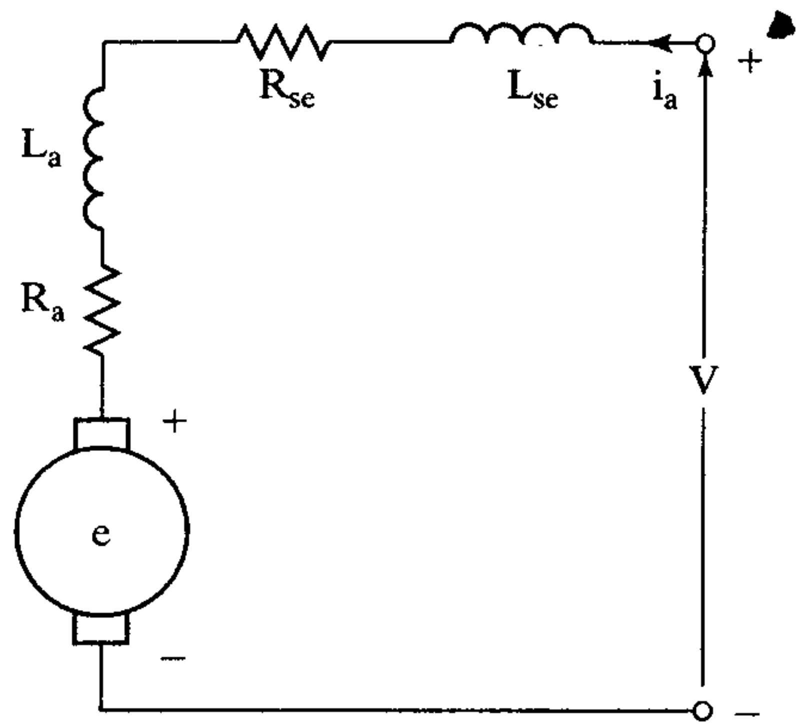

Figure 2.7 dc series machine

- DC compound machine

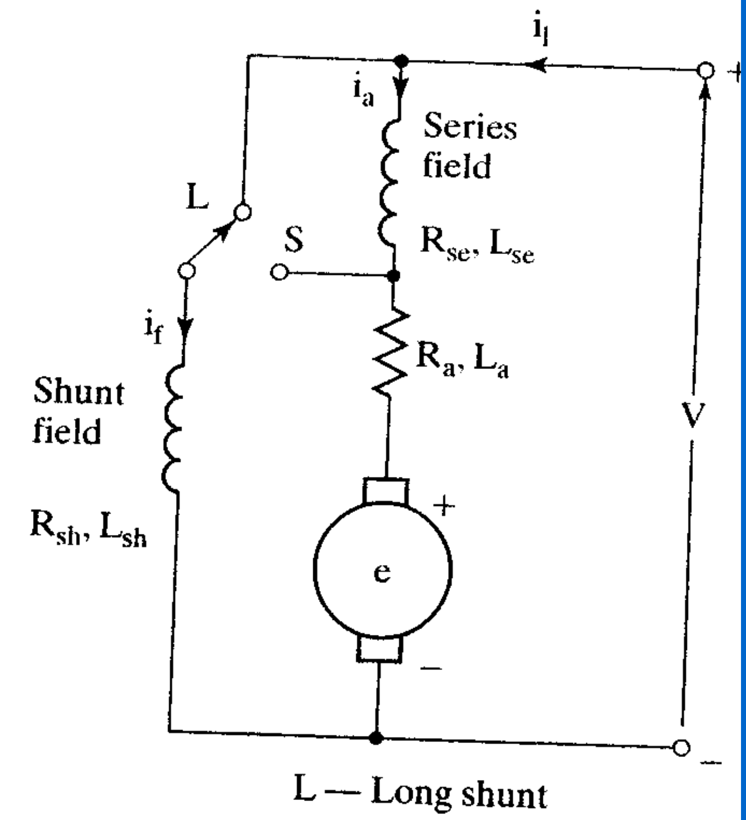

Figure 2.9 DC compound machine

• Permanent-magnet dc machine

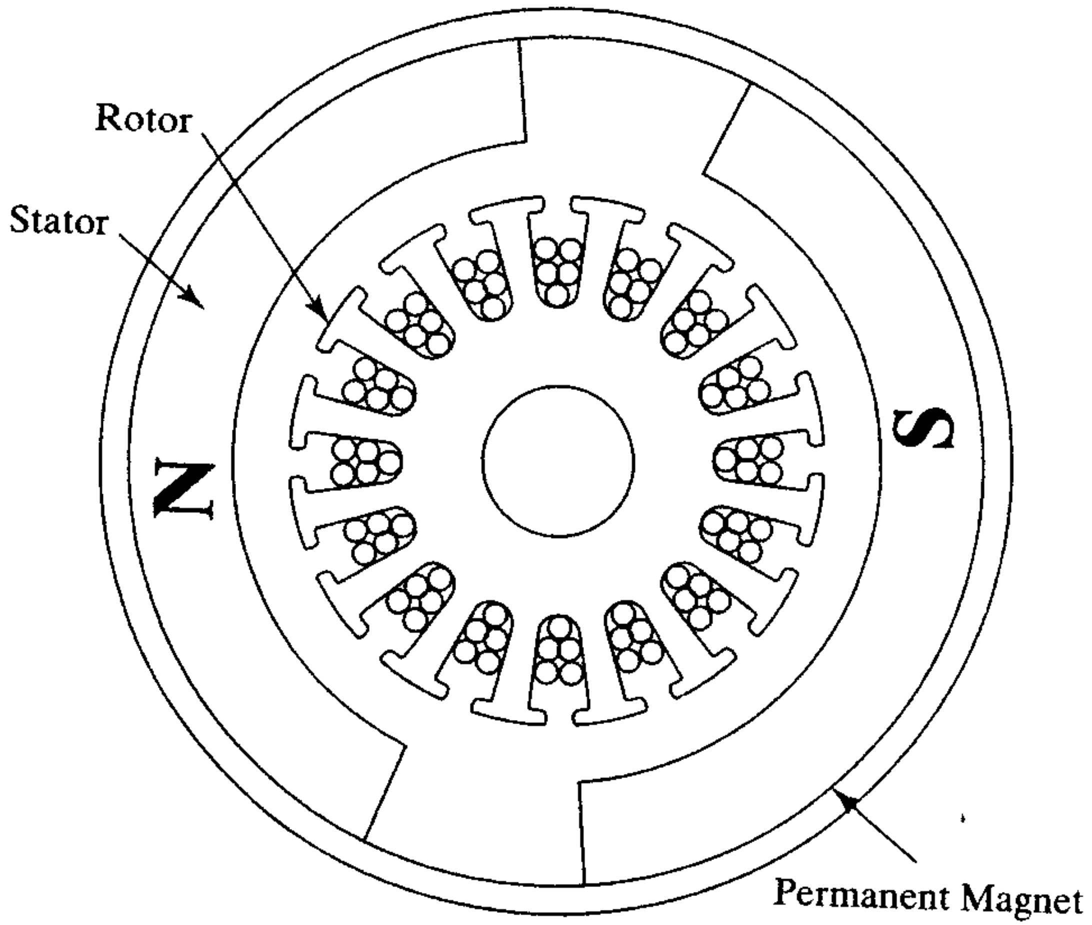

Figure 2.10 Cutaway view of a conventional permanent-magnet dc motor assembly

2.8 Measurement of motor constants

- Armature resistance:

  It is measured between the armature terminals by applying a dc voltage. (need to subtract the brush and contact resistance)

- Armature inductance:

  By the test schematic shown in Figure 2.11, the inductance is

$$L_a = \frac{\sqrt{\frac{V_a^2}{I_a} - R_a^2}}{2\pi f_s}$$

f_s: the frequency

R_a: the armature ac resistance

- EMF constant

Specified field voltage is applied and kept constant, and the shaft is rotated by another dc motor, and then the armature is connected a voltmeter.

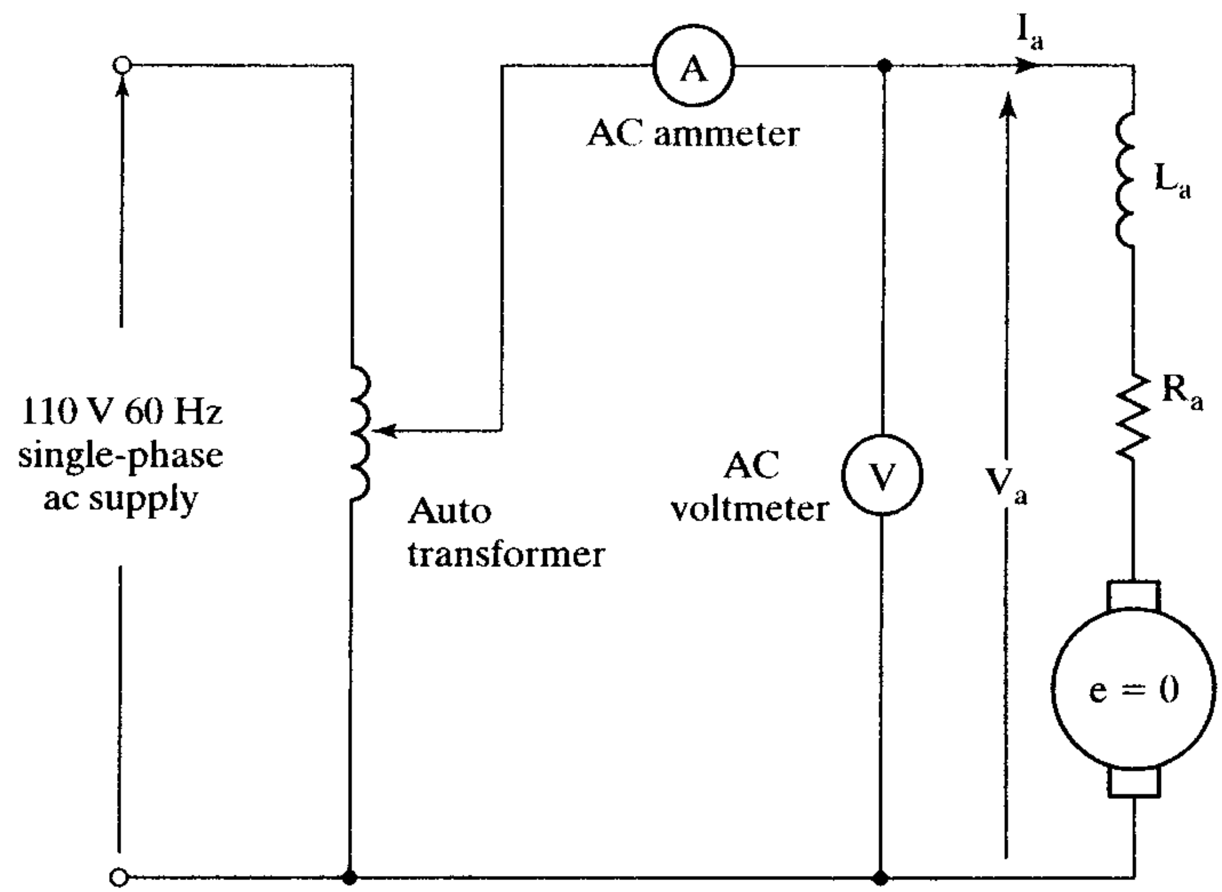

Figure 2.11 Measurement of armature inductance