Basic Inventory Models

- Inventory Concepts
- Q models and P models
- EOQ
- Variations of EOQ model
- Trade-off between Inventory and Service

Inv1-1

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1. Inventory Concepts

Manufacturing/Transportation
- raw materials 製程所需的原料與外購零件
- **W**ork **I**n **P**rocess 在製程中等待處理的半成品
- **F**inished **G**oods **I**nventory 等待出貨的成品
- spare parts and supply 維持系統運作的消耗性零件

Retail/Service
- merchandise 銷售給顧客的商品
- service supply 進行服務所需物料，如購物袋、碗盤刀叉

Inv1-2

Rationale for Inventory

Batching Economies/Cycle Stocks
>   Price discounts 採購價格的優惠
>   Transportation rate discounts 運費的優惠
>   Production economics 生產效率的考量

Uncertainty/Safety Stocks
>   保障原料與零組件不立即斷料
>   使生產排程獲得緩衝
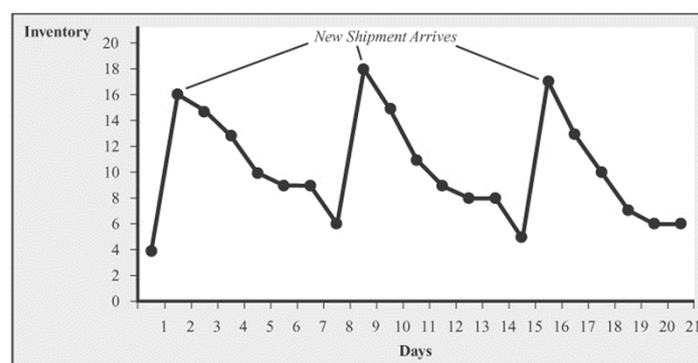>   應付市場需求的變化

In-Transit (pipeline) and Work-in-Process Stocks
>   節省運費、降低生產進度不協調的影響

Seasonal Stocks   季節性的供應

Anticipatory Stocks   旺季或漲價前的囤積

Inv1-3

Cycle Inventory

採購價格的優惠、運費的優惠、生產效率的考量

Inv1-4

Seasonal Inventory

甜菜的收成期為六週，消耗庫存需要長期時間

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Inventory costs: 20% to 40% of the inventory's worth

● Holding or Carrying Cost (包含運輸途中的庫存)
capital (opportunity) cost 資金積壓所損失的利息或投資利潤
storage space cost 倉儲空間與設備的成本
operation cost 人事費用、水電費、稅金
risk cost 貶值、保費、偷竊或毀損

● Ordering or Setup Cost
對外採購或內部製造所需的前置作業成本

● Shortage Costs, Backordering, Lost Sales
停工的損失、延誤交貨的罰款、損失的利潤

Inv1-6

Inventory Decision Making

Decisions: When to order? How many to order?

- Product Characteristics 產品特性

- Demand Variability 需求變異

- Service Level Requirement 服務要求

- Replenishment Lead Time 補貨時間

- Ordering Cost and Holding Cost 訂貨與持有成本

- Stock out or Backordering Cost 缺貨與欠貨成本

Inv1-7

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2. Q models and P models
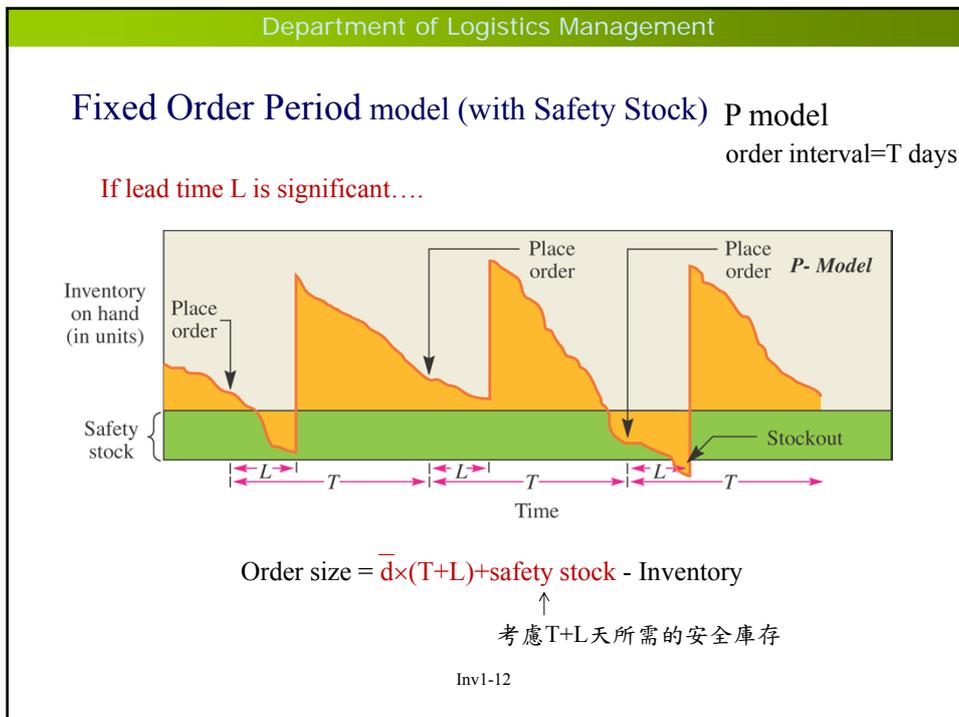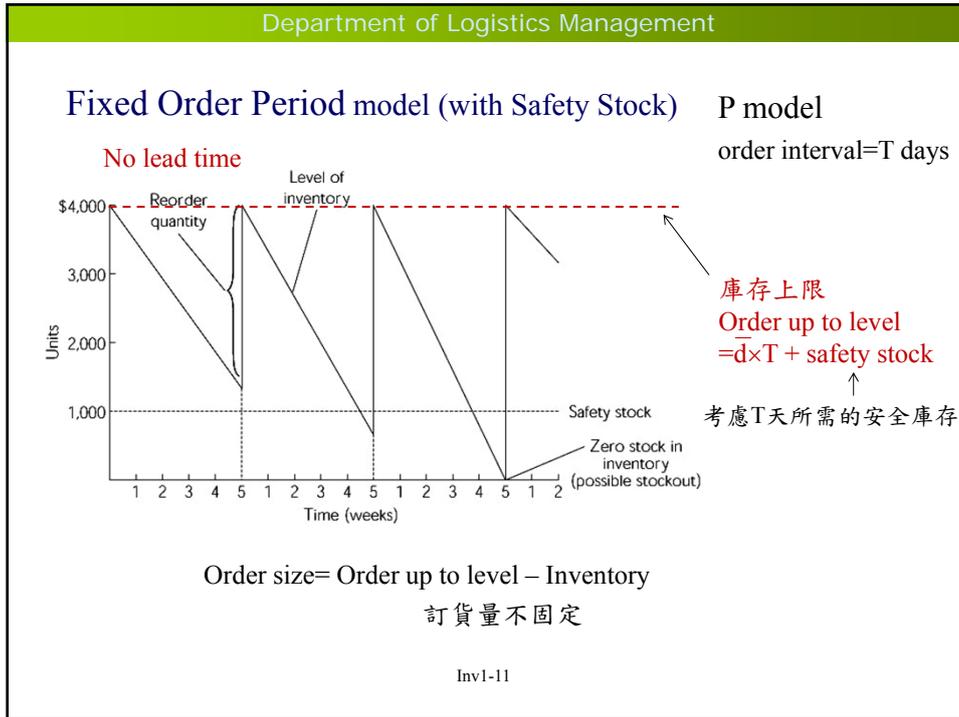
假設需求沒有趨勢或季節性因素，只有隨機變動。
⇒可根據過去銷售計算未來的日平均需求與需求標準差

Question: 如何補充庫存以滿足平日的需求？

Answer:

Fixed Order Quantity model (Q model)

Fixed Order Period model (P model)

Inv1-8

Fixed Order Quantity model   Q model (s, S)=(R,Q)

S = 庫存上限(帳面) s=訂貨點 L=lead time

Order quantity Q≡S-s   Reorder point $R=\bar{d}\times L$

Average Inventory =Q/2

單日平均需求

Inv1-9

Fixed Order Quantity model
under the condition of Uncertainty

訂貨間隔時間不固定

Order quantity=Q

Reorder point $R=\bar{d}\times L$ + safety stock

Average Inventory
=Q/2+safety stock

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Fixed Order Period model (with Safety Stock) P model

order interval=T days

No lead time

庫存上限
Order up to level
=d×T + safety stock

考慮T天所需的安全庫存

Order size= Order up to level – Inventory

訂貨量不固定

Inv1-11

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Fixed Order Period model (with Safety Stock) P model

order interval=T days

If lead time L is significant….

Order size = $\overline{d}$×(T+L)+safety stock - Inventory

考慮T+L天所需的安全庫存

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以Excel模擬固定週期庫存管理

H5	▼		fx	=MAX(H$2-(E5+F5+G5),0)								
B	C	D	E	F	G	H	I	J	K	L	M	N
						庫存上限	單位利潤	單位持有	單位缺貨	檢查成本	訂貨成本	
						400	$5	$1	$2	$30	$70	
期初庫存	顧客需求	銷售量	期末庫存	在途1	在途2	訂貨量	銷售利潤	持有成本	缺貨損失	檢查成本	訂貨成本	盈餘
400	213	213	187	0	0	213	$1,065	$293.50	$0	$30	$70	$672
187	179	179	8	213	0	179	$895	$310.50	$0	$30	$70	$485
8	182	8	0	179	213	8	$40	$396.00	$348	$30	$70	($804)
213	177	177	36	8	179	177	$885	$311.50	$0	$30	$70	$474
215	282	215	0	177	8	215	$1,075	$292.50	$134	$30	$70	$549
8	218	8	0	215	177	8	$40	$396.00	$420	$30	$70	($876)
177	255	177	0	8	215	177	$885	$311.50	$156	$30	$70	$318
146	267	146	0	134	120	146	$730	$327.00	$242	$30	$70	$61
120	201	120	0	146	134	120	$600	$340.00	$162	$30	$70	($2)
134	176	134	0	120	146	134	$670	$333.00	$84	$30	$70	$153
146	161	146	0	134	120	146	$730	$327.00	$30	$30	$70	$273
120	128	120	0	146	134	120	$600	$340.00	$16	$30	$70	$144
148.46	183.23	129.48	18.98				$647.40	$335.26	$107.50			$104.6

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Two-Bin System   Option Replenishment

demand 1st bin 2nd bin

demand 1st bin 2nd bin

1st bin 2nd bin

------► replenishment

等候需求 demand
領料/出貨
No 週期 **T** 已至
Yes
No 庫存量I < **r**
訂貨量≥M-**r**
Yes
訂貨量=M-I

Inv1-14

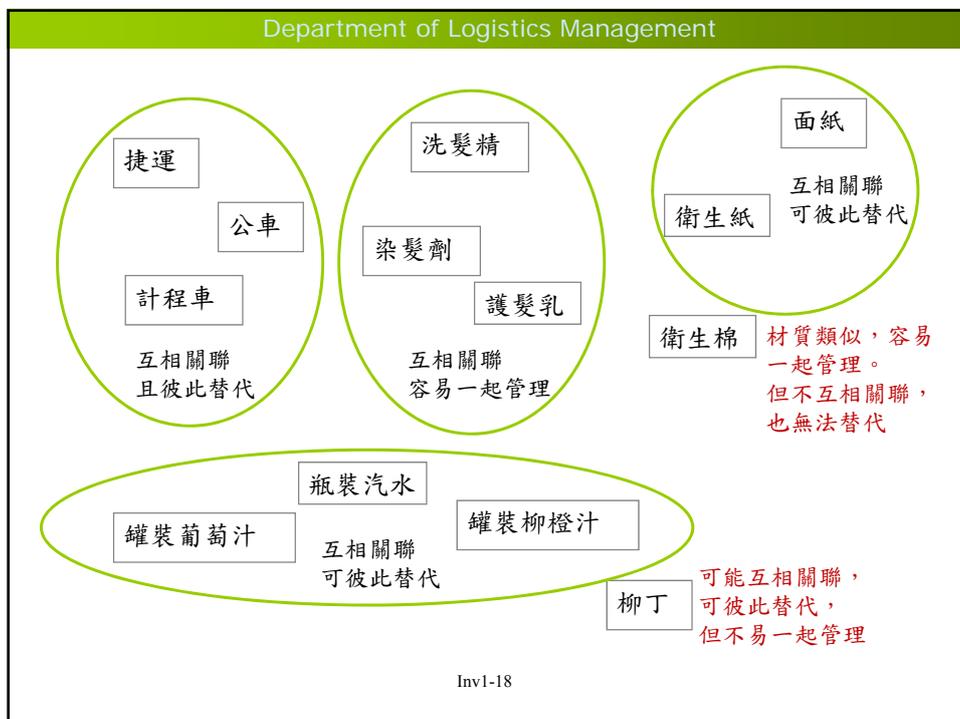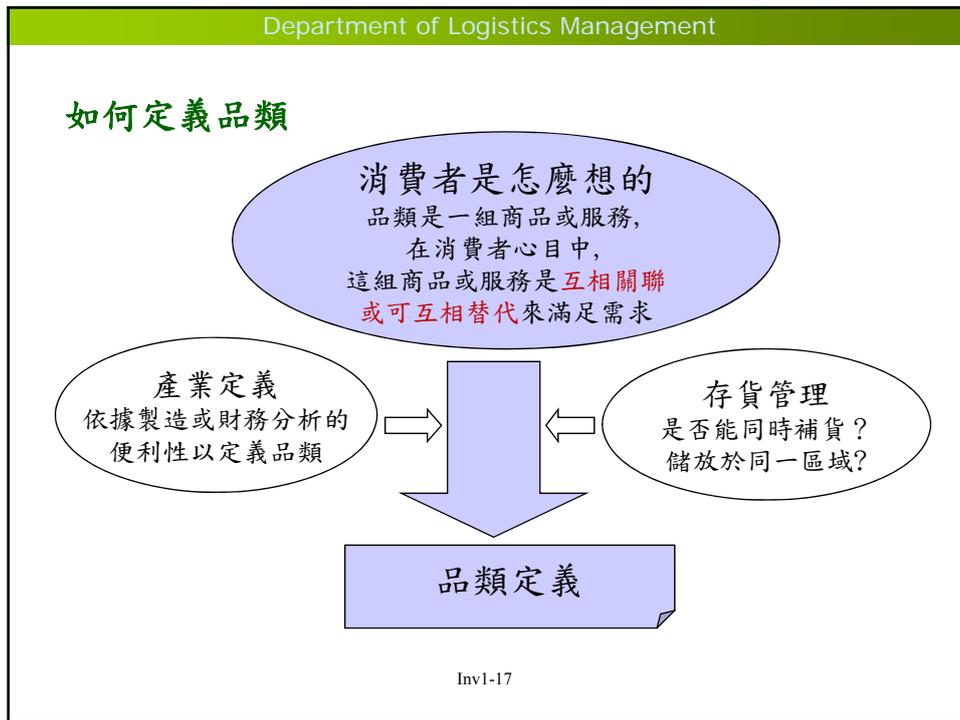# 3. Inventory Control for ABC Classification

- Class A=large volume or low volume but high value.
- Use Q models with Normal demand for fast-moving A items.
- Adjust reorder point (safety stock) according to supply condition to control shortage. 控制訂貨點(安全庫存)以避免缺貨
- Supplier capacity and delivery delay can cause problems. 預防供應商產能不足或運輸延誤(lead time)

- Business may carry a hi-end item for its prestige value. 旗艦≠暢銷
- Lot for Lot ordering can reduce holding cost.
- Use Basestock models (s=S-1, Q=1) for very expensive A items.

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Managing Routine (Class B, C) Inventories

1. Items with low dollar volume need not be placed in the class C. 需考慮庫存管理的整體成本以決定B或C

2. Shortages on a slow moving product can cause severe reductions in the usage of class A items. C類缺貨可能影響A類銷售

3. Shortages on a cheap components can cause the delay of the complete product and force customers switching to competitors. 小零件釀成大問題

4. Use P models or two-bin systems for inventory control.

5. Group into families to reduce cost. 品類管理 Category Management

6. Review the ABC classification periodically.

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如何定義品類

消費者是怎麼想的
品類是一組商品或服務,
在消費者心目中,
這組商品或服務是互相關聯
或可互相替代來滿足需求

產業定義
依據製造或財務分析的
便利性以定義品類

→ ←

存貨管理
是否能同時補貨?
儲放於同一區域?

品類定義

Inv1-17

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捷運
公車
計程車

互相關聯
且彼此替代

洗髮精
染髮劑
護髮乳

互相關聯
容易一起管理

面紙

互相關聯
可彼此替代

衛生紙

衛生棉
材質類似,容易
一起管理。
但不互相關聯,
也無法替代

瓶裝汽水
罐裝葡萄汁
罐裝柳橙汁

互相關聯
可彼此替代

柳丁
可能互相關聯,
可彼此替代,
但不易一起管理

Inv1-18

Stocking vs. Not Stocking a Class C Item

Many class C items with extremely low sales may be eliminated from catalog to save operating cost. 實體店面空間有限

Avoid stocking a class C item if
- The operating cost is too high compared to its value. 整體成本超過貢獻
- High depreciation and/or carrying cost. 持有成本高
- Low ordering or setup cost and fast delivery. 取得快速方便
- Customers are willing to wait. 缺貨成本低
- An internal supplier (central warehouse) is available.

Some items may be variations of popular items and should be kept in stock due to marketing considerations. 因行銷考量而保持產品線完整

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4. Economic Order Quantity

Fixed Order Quantity Approach (Condition of Certainty)

- D = annual rate of demand
- Q = order quantity (production lot size)
- S = order or setup cost
- c = value or cost of one unit in dollars
- α = carrying cost per dollar value in percent
- $h = \alpha c$ = annual storage cost in \$/unit 與單價成正比
- TC = total costs in dollars per year

Inv1-20

Economic Order Quantity

$$TC(Q) = \frac{D}{Q}S + \frac{Q}{2}h \cdot 1$$

$$\frac{d\,TC}{d\,Q} = 0 \Rightarrow Q^* = \sqrt{\frac{2DS}{h}}$$

全年庫存成本=訂貨成本+持有成本
（不考慮採購價格）

訂貨時間間隔

$$T^* = \frac{1}{D/Q^*} = \sqrt{\frac{2S}{Dh}}$$

d=單日需求　　L=交貨天數

$\Rightarrow$ 訂貨點=d×L

Inv1-21

Optimality=Balance

$$\frac{D}{Q^*}S \qquad \frac{Q^*}{2}h \cdot 1$$

| Q	Order Cost	Holding Cost	Total Cost
100	$7,200	$1,250	$8,450
140	5,143	1,750	6,893
180	4,000	2,250	6,250
220	3,273	2,750	6,023
240	3,000	3,000	6,000
260	2,769	3,250	6,019
300	2,400	3,750	6,150
340	2,118	4,250	6,368
400	1,800	5,000	6,800
500	1,440	6,250	7,690

Inv1-22

Insights of EOQ

If $Q = 1.5 \times Q^*$

Then $TC(Q) = \dfrac{D}{Q}S + \dfrac{Q}{2}h \cdot 1 \approx 1.08 \cdot TC(Q^*)$

⇒可微調訂貨量，使品項的訂貨時間間隔配合整體的採購與進貨作業

Powers of 2 Order Intervals

$T=2^m$ $T=2^{m+1}$

2^m $2^m\sqrt{2}$ 2^{m+1}

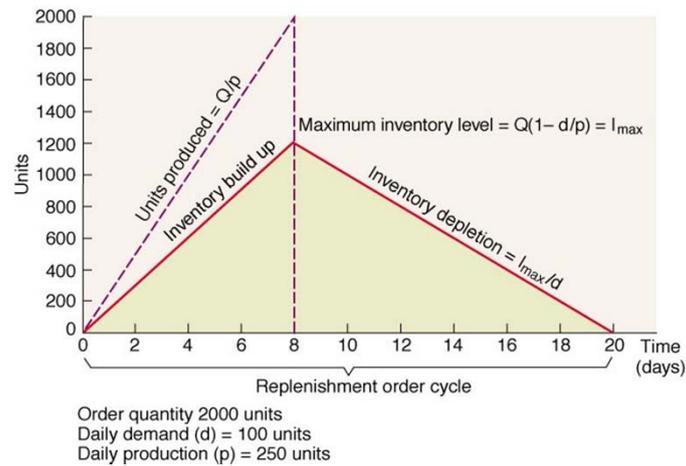2 weeks   2.83 weeks   4 weeks
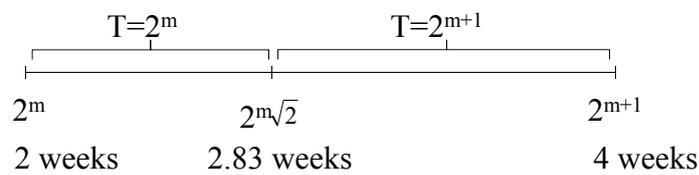
Inv1-23

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5. Relaxing Assumptions of EOQ Model

- Fixed setup costs
- Fixed unit price
- Instantaneous delivery/production
- Long shelf life

- Deterministic and constant demand
- Fixed delivery lead time

Inv1-24

EOQ and Setup Costs

$$Q* = \sqrt{\frac{2DS}{h}}$$

Alternative Thinking: reducing S
$\Rightarrow$ reducing Q* and Total cost

1950s 美國年銷售量800萬輛汽車 vs. 豐田每月賣出兩千輛

1953 日本被迫開放汽車市場，大野耐一負責降低成本、改善品質

In 1990s, Ford carried 15 times as much WIP as Toyota

換線作業   外部轉換   內部轉換   作業改善   內部轉換   設備改善   內部轉換

0   1   2   3   4

EOQ and Unit Price: price break models

Annual Demand (D)=10,000 units   Carrying cost (α) = 20% of total cost
Cost to place an order (S)=$20   Cost per unit ($c$) = $5.0 (<500)
                                                = $4.5 (<1000)
                                                = $3.9 ($\geq$1000)

$$TC(Q) = Dc + \frac{D}{Q}S + \frac{Q}{2}\alpha c \cdot 1$$

@ C = $4.50
@ C = $5.00
@ C = $3.90

Ordering cost and holding cost

1,100 1,000 900 800 700 600 500 400 300 200 100

100 200 300 400 500 600 700 800 900 1,000 1,100 1,200 1,300

Units

633   666   716

EOQ for Production: Economic Production Quantity

補貨時緩慢增加庫存，如生產零組件或供應商分批交貨

Order quantity 2000 units
Daily demand (d) = 100 units
Daily production (p) = 250 units

EPQ Decisions

p: daily production rate d: daily demand rate

Maximum Inventory: $I_{MAX} = Q\left(1 - \dfrac{d}{p}\right)$

EPQ Total Cost: $TC_{EPQ} = \left(\dfrac{D}{Q}S\right) + \left(\dfrac{I_{MAX}}{2}h \cdot 1\right)$

$$EPQ = \sqrt{\dfrac{2DS}{h\left(1 - \dfrac{d}{p}\right)}}$$

Inv1-28

EOQ with Perishable Products

1. 產品進貨後只有U天的貨架壽命 $\Rightarrow Q \leq d \times U$

EOQ model $Q^* \leq d \times U \Rightarrow$ 最佳訂貨量不變

EOQ model $Q^* > d \times U \Rightarrow$ 最佳訂貨量降為 $d \times U$

2. 產品進貨後，每年約有 δ 的損毀比率

EOQ model Q^* 略降為 $\sqrt{\dfrac{2DS}{(h+\delta c)}}$

Inv1-29

6. Safety Stock

In reality, demand is not constant but varies from day to day.

- amount of inventory carried in addition to expected demand E(D)
- provide protection against stockout before the replenishment
- 需考慮缺貨風險時間長度、需求變異性、缺貨成本

P (demand>E(D)+safety stock)

Probability (Stockout)

E(D)
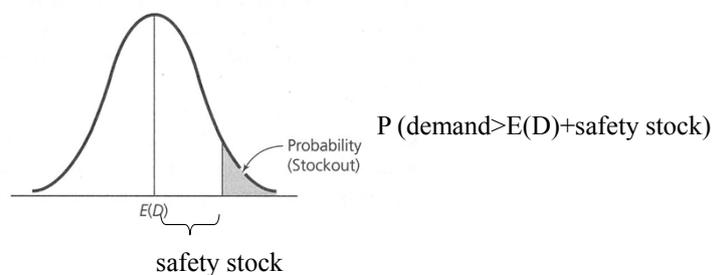
safety stock

Inv1-30

Demand During Lead Time L

如果需求沒有趨勢或尖離峰變化，可用過去紀錄計算

Daily demand $\sim N(\overline{d}, \sigma_d^{\,2})$ $\quad \sigma_d^{\,2} \approx \dfrac{\sum (d_i - \overline{d})^2}{n-1}$

Demand during lead time $\sim N(L \times \overline{d}, L \times \sigma_d^{\,2})$
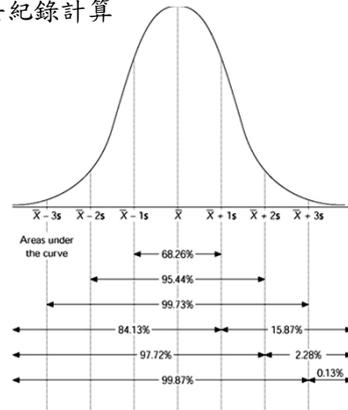
如果未來需求將有明顯變化，需要預測交貨期間的平均需求D_L與變化範圍 $\sigma_L \Rightarrow N(D_L, \sigma_L^{\,2})$

（常態分配圖，Areas under the curve: 68.26%, 95.44%, 99.73%, 84.13%, 15.87%, 97.72%, 2.28%, 99.87%, 0.13%）

Inv1-31

Fixed Order Quantity with Safety Stock

Decision: 訂貨點R

缺貨風險時間長度$=L$

$$R = D_L + z \times \sigma_L$$

$D_L =$交貨期間的平均需求

$\sigma_L =$交貨期間的需求標準差 $= \sqrt{L \cdot \sigma_d^{\,2}}$

z =service level
庫存不缺貨之機率 $\geq 95\%\ (99\%) \Rightarrow$ z=1.64 (2.33)

（圖：Q, R, B, Range of demand, Safety stock, Stockout, Time, L, 0）
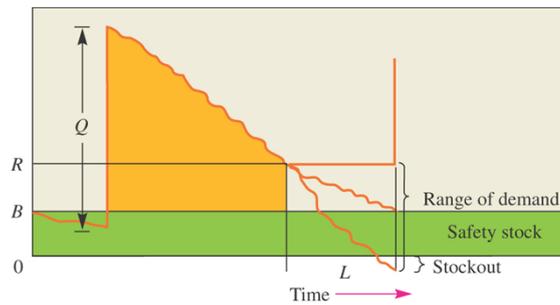
Inv1-32

Fixed Time Period with Safety Stock

Decision
訂貨量Q

缺貨風險時間長度
$=T+L$
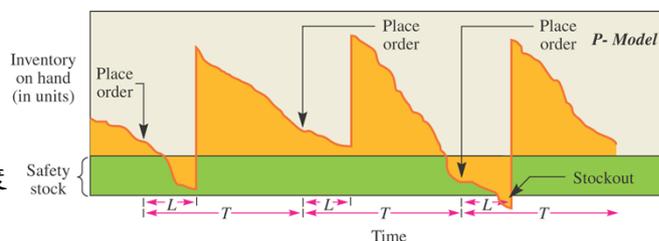
庫存總需求=Average demand + Safety stock

$= \bar{d}(T+L) + \boxed{z \times \sigma_{T+L}}$ =現有庫存量I +訂貨量Q

Safety stock $= z \times \sigma_{T+L}$

$\sigma_{T+L}{}^2 \approx (T+L) \times \sigma_d{}^2 \approx (T+L) \times \dfrac{\sum(d_i - \bar{d})^2}{n-1}$

Inv1-33

採購前置期

無論訂貨數量多少，從訂單發出到接收物品有一個延遲的
時間，稱為採購前置期(Purchase lead time)。其運算式為
L=T1 +T2 +T3 +T4 +T5

- T1 ——企業內部的訂貨準備時間；
- T2 ——訂單傳送時間；
- T3 ——供應廠家準備物品的時間；
- T4 ——運輸的時間；
- T5 ——入庫前對物品進行驗收等活動所佔用的時間。

除T1 和T5 可控外，其他變數都是不可控的。

Inv1-34

Uncertainty of Lead Time

Daily demand ~ $N(\bar{d}, \sigma_d^2)$ Lead time ~ $N(L, \sigma_L^2)$

Demand during lead time ~ $N(L \times \bar{d}, \sigma^2)$ $\sigma^2 = L\sigma_d^2 + \bar{d}\,\sigma_L^2$

Q model $R = D_L + z \times \sqrt{L\sigma_d^2 + \bar{d}\,\sigma_L^2}$

$\Rightarrow$ 交期不穩定會增加缺貨風險，對策是增加安全庫存

Alternative Thinking: reducing $L \Rightarrow$ reducing safety stock

Inv1-35

7. Other EOQ Models: Q model with Backorders

D=年度需求 S=訂貨成本 h=單位持有成本 b=單位欠貨成本
Q=訂貨批量   R=訂貨點   I=平均庫存   B=平均欠貨量

$$TC(R,Q) = \frac{D}{Q}S + I(R,Q)h \cdot 1 + B(R,Q)b \cdot 1$$

$$Q^* = \sqrt{\frac{2DS}{h}}$$

Optimal service level $\Phi(z) = \dfrac{b}{b+h} \Rightarrow R^* = \bar{d}L + z\sigma_L$

$b=h \Rightarrow z=0 \Rightarrow$ No safety stock

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Inventory Rationing

- 不同類型顧客的利潤不同，因而有不同缺貨成本
- Q model: 固定訂購量Q，訂貨點R，保留庫存量X
- 當庫存降至X，停止出貨給低利潤的顧客。
- 當補貨到達，先出貨給高利潤的顧客。

- 保留量X設定太高，增加持有成本與流失低利潤顧客。
- 保留量X設定太低，流失高利潤顧客。

Inv1-37

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$$Q^* = \sqrt{\frac{2DS}{h}} \qquad R = D_L + z \times \sigma_L \qquad \Phi(z) = \frac{b}{b+h}$$

Constant Demand $\Rightarrow$ No safety stock is required.

Zero Lead Time $\Rightarrow$ Reorder point=0

No Ordering Cost $\Rightarrow$ EOQ=0 $\Rightarrow$ JIT Delivery

No Backordering Cost $\Rightarrow$ z = -∞ $\Rightarrow$ Wait until backorders = EOQ

Inv1-38

8. Performance Measures of Inventory Control

inventory turnover

weeks of supply

$$庫存週轉率 = \frac{全年銷售金額}{平均庫存價值}$$

$$供應週數 = \frac{平均庫存價值}{全年銷售金額} \times 52$$

service level

$$fill\ rate = \frac{如期供貨量}{總需求量}$$

$$服務水準 = \frac{如期供貨次數}{總訂貨次數}$$

Inv1-39

Saving Inventory Dollars by Inventory Turns

TABLE 6–8 The Relationship among Inventory Turnover, Average Inventory, and Inventory Carrying Costs

Inventory Turnover	Average Inventory	Inventory Carrying Cost*	Incremental Savings in Carrying Cost	Cumulative Savings in Carrying Cost
1	$20,000,000	$6,000,000	–	
2	10,000,000	3,000,000	$3,000,000	
3	6,666,667	2,000,000	1,000,000	
4	5,000,000	1,500,000	500,000	
5	4,000,000	1,200,000	300,000	
6	3,333,333	1,000,000	200,000	
7	2,857,143	857,143	142,857	
8	2,500,000	750,000	107,143	
9	2,222,222	666,667	83,333	
10	2,000,000	600,000	66,667	

*Assume that inventory carrying cost equals 30%.

Inv1-40

Trade-off between Inventory and Customer Service
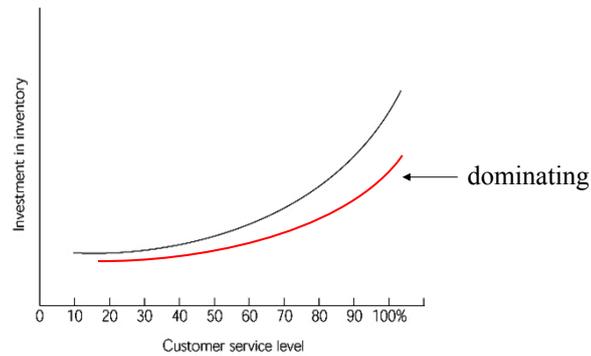
Are customers satisfied with the current level of service?

可跟客戶商定service level (fill rate)，然後努力降低成本

Inv1-41

SUMMARY

- There are reasons for carrying inventory.

- Inventory decisions: how much to order and when to order

- EOQ: ordering cost vs. holding cost

- Safety Stock: variability affects service level

- Customer service: Inventory cost vs. Backorder cost

- Manage trade-off between inventory cost and service

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