Supplement A: Break-even analysis

- Break-even analysis 損益平衡分析
  - Analysis to compare processes by finding the volume at which two different processes have equal total costs.
- Break-even quantity 損益平衡點
  - The volume at which total revenues equal total costs.

Financial Considerations

- **Unit variable cost (*c*)** cost per unit for materials, labor and etc.
- **Fixed cost (*F*)** the portion of the total cost that remains constant regardless of changes in levels of output.
- **Quantity (*Q*)** the number of customers served or units produced per year.

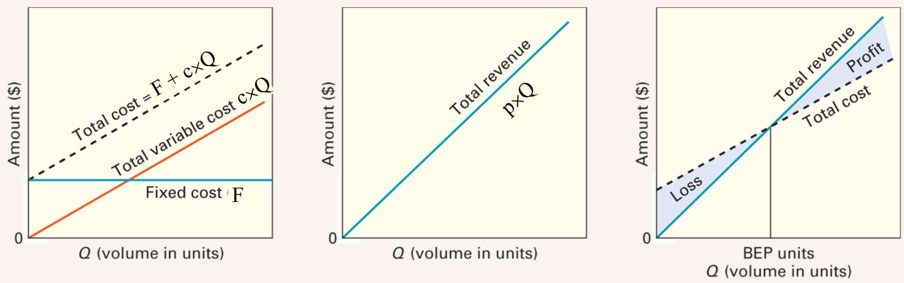
- Total Cost = Fixed Cost + Total Variable Cost = $F + c \times Q$
- Total Revenue = unit revenue (p) $\times$ Quantity (Q)
- Total Profit = $p \times Q - (F + c \times Q)$

Break-Even Analysis

Total Profit = p × Q – (F + c × Q)

Break-even quantity

Total Revenue = Total Cost ⇒ p × Q = (F + c × Q) ⇒ $Q = \dfrac{F}{p-c}$



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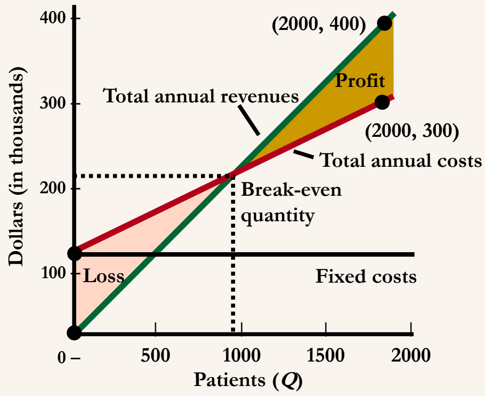
Example A.1

A new procedure will be offered at $200 per patient. The fixed cost per year would be $100,000 with variable costs of $100 per patient. What is the break-even quantity for this service?

$$Q = \frac{F}{p-c} = \frac{100{,}000}{200-100}$$

$= 1{,}000$ patients



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Financial Analysis

- 水餃店原本雇用兼職人員包水餃，時薪$150。
- 現在考慮購買包水餃機以取代人工。
- 機器雙人操作，每小時可達600個水餃
- Consider time value of money, present value=$150,000
- Payback period=5 years
- Annual interest rate=5%
- Annual net cash flow=
  =PMT(5%,5,150000,0)
  =$34646

NPV 計算淨現值

Evaluating Alternatives: Make or Buy

- F_b : The fixed cost (per year) of the buy option
- F_m : The fixed cost of the make option
- c_b : The variable cost (per unit) of the buy option
- c_m : The variable cost of the make option

  - Total cost to buy = $F_b + c_b \times Q$
  - Total cost to make = $F_m + c_m \times Q$

$$F_b + c_b \times Q = F_m + c_m \times Q \qquad \Rightarrow Q = \frac{F_m - F_b}{c_b - c_m}$$

Example A.3

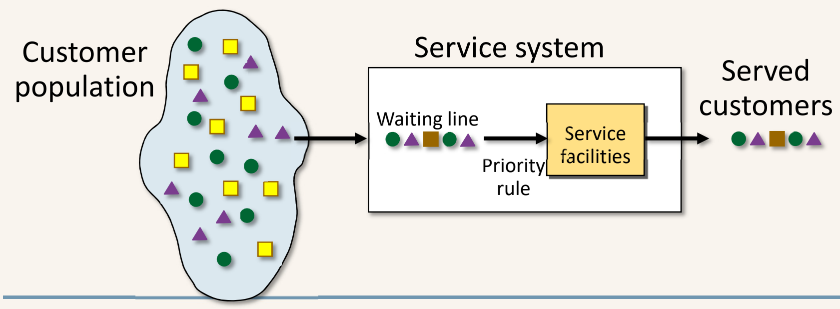
- A fast-food restaurant is adding salads to the menu.
- Make ⇒ Fixed costs: $12,000, variable costs: $1.50 per salad.
- Buy ⇒ Preassembled salads could be purchased from a local supplier at $2.00 per salad. It would require additional refrigeration with an annual fixed cost of $2,400
- The price to the customer will be the same.
- Expected demand is 25,000 salads per year.

$$Q = \frac{F_m - F_b}{c_b - c_m} = \frac{12{,}000 - 2{,}400}{2.0 - 1.5} = 19{,}200 \text{ salads} \quad < 25{,}000$$

Supplement B: Waiting Line Models

Q: What are waiting lines and why do they form?

A: Waiting Lines form due to a **temporary imbalance** between the demand for service and the capacity of the system to provide the service. 顧客異質性使供需短暫失調



Structure of Waiting-Lines

1. An input, or *customer population*, that generates potential customers (single channel vs. multiple channels)
2. A *waiting line* of customers (號碼牌)
3. The *service facility*, consisting of a person (or crew), a machine (or group of machines), or both necessary to perform the service for the customer
4. A *priority rule*, which selects the next customer to be served by the service facility (FCFS)
5. Single phase vs. multiple phase

Random Arrivals

Poisson arrival distribution

$$P_n = \frac{(\lambda T)^n e^{-\lambda T}}{n!} \quad \text{for } n = 0, 1, 2, \ldots$$

P_n =Probability of n arrivals in T time periods

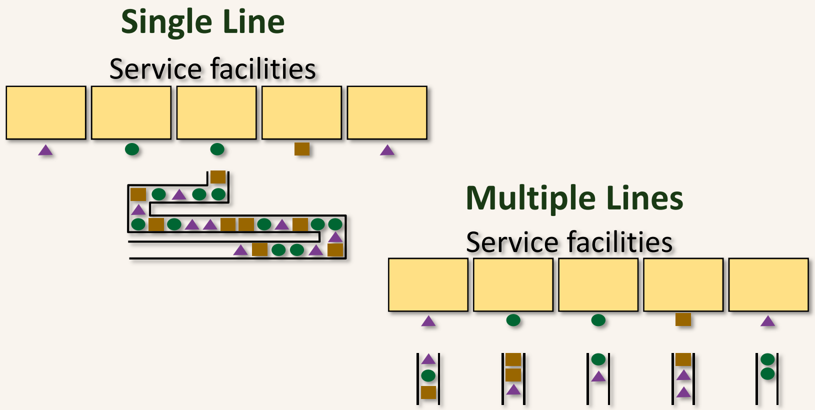
λ = Average numbers of arrivals per period 無尖離峰變化

e = 2.7183

λ = 2 arrivals per hour, T = 1 hour, and n = 4 arrivals.

$$P_4 = \frac{[2(1)]^4 e^{-2(1)}}{4!} = \frac{16\, e^{-2}}{24} = 0.090$$

Waiting Line Arrangements



Priority Rules

- First-come, first-served (FCFS)
- Earliest due date (EDD)
- Shortest processing time (SPT)
- Preemptive discipline (emergencies first)

Customer Service Times

Exponential service time distribution

$$P(t \leq T) = 1 - e^{-\mu T}$$

μ = average number of customer completing service per period

t = actual service time of the customer

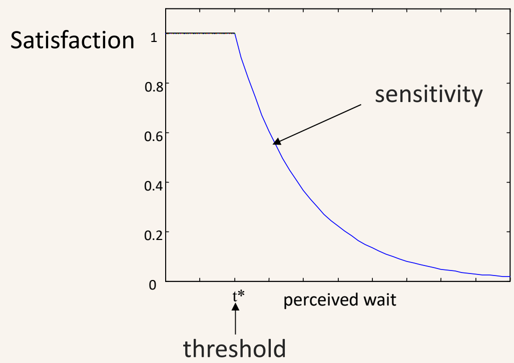
T = target service time

μ = 3 customers per hour, T = 10 minutes = 0.167 hour.

$$P(t \leq 0.167 \text{ hour}) = 1 - e^{-3(0.167)} = 1 - 0.61 = 0.39$$

Waiting-Line Models to Analyze Operations

- Balance costs (capacity, lost sales) against benefits (customer satisfaction)



Single-Line Single-Server Model 單人服務

- Single-server, single waiting line, and only one phase
- Assumptions are:
  1. Customer population is infinite and patient
  2. Customers arrive according to a Poisson distribution, with a mean arrival rate of λ
  3. Service distribution is exponential with a mean service rate of μ
  4. Mean service rate exceeds mean arrival rate $\lambda < \mu$
  5. Customers are served FCFS
  6. The length of the waiting line is unlimited

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Single-Line Single-Server Model

ρ = Average utilization of the system = $\frac{\lambda}{\mu} < 1$

P_n = Probability that n customers are in the system = $(1-\rho)\rho^n$

L = Average number of customers in the system = $\frac{\lambda}{\mu-\lambda}$

L_q = Average number of customers in waiting = $L - \rho = \rho L$

W = Average time spent in the system, including service = $\frac{1}{\mu-\lambda}$

W_q = Average waiting time in line = $W - \frac{1}{\mu} = \rho W$

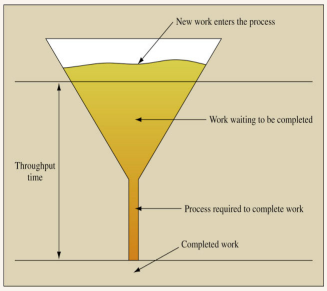
Little's Law

A fundamental law that relates the number of customers in a waiting-line system to the arrival rate and average time in system

λ = arrival rate 流速

average time in system $W = \dfrac{L \text{ customers}}{\lambda \text{ customer/hour}}$

Work-in-process $L = \lambda W$



Single-Line Multiple-Server Model 平行服務

- Service system has only one phase, multiple-channels
- Assumptions (in addition to single-server model)
  - There are ***s*** **identical** servers
  - Exponential service distribution with mean $1/\mu$
  - $s\mu$ should always exceed λ

◉ Single-server model ○ Multiple-server model ○ Finite-source model

Servers		(Number of servers is assumed to be 1 in single-server model.)
Arrival Rate (λ)	30	
Service Rate (μ)	35	

Probability of zero customers in the system (P_0)	0.1429
Probability of exactly 0 customers in the system (P_n)	0.1429
Average utilization of the server (ρ)	0.8571
Average number of customers in the system (L)	6.0000
Average number of customers in line (L_q)	5.1429