

Chapter 9 Managing Inventory

- Types of Inventory
- ABC Analysis
- Q models and P models
- Supplement C: Special Inventory Models

Inventory Management at



- 44M streaming customers worldwide and 7M DVD customers in US.
- 89M discs distributed across 39 warehouses in US. Each can process 60000 orders per day.
- Discs are scanned by machines to see if any customer has ordered them. Others are shelved for future rentals.
- Proper managed inventories can support competitive priorities of **variety** and **delivery speed**.

What is Inventory?

A stock of materials used to satisfy customer demand or to support the production of services or goods.

Manufacturing Inventory

raw materials, WIP, maintenance and repairs, supplies, FGI

Retail Inventory

merchandise, supplies

餐飲業？

What is a Inventory Management?

The **planning** and **controlling** of inventories to meet the competitive priorities of the organization.

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Pressures for Small Inventories

Inventory holding cost

- Cost of capital (opportunity cost)
- Storage and handling costs: space, labor, machines
- Taxes 各國稅法不同, Insurance
- Shrinkage
 - Pilferage by customers or employees
 - Obsolescence due to model changes or new products
 - Deterioration: limited shelf life

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Pressures for Large Inventories

- Customer service: stockout (lost sale) or backorder
- Ordering cost: fixed cost of preparing a purchase order
- Transportation cost: full truckload or full container
- Payments to suppliers (quantity discount) 採購↔倉管
- ◆ Setup cost: fixed cost of changing over a machine to produce a different product.
- ◆ Labor and equipment utilization

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Inventory Costs

- Purchase Cost
- Holding or Carrying cost ← order too much or too early
 - opportunity cost 資金積壓的潛在損失
 - storage cost 倉儲設備、進出盤點
 - shrinkage cost 貶值、偷竊、毀損、保費
- Ordering or Setup cost 前置作業成本 ← order too often or too little
 - 對外採購：聯繫、運輸、驗收
 - 內部製造：停機調整、試產
- Shortage costs or Lost Sales ← order too little or too late
 - 停工減產的損失、延誤交貨的罰款
 - 銷售減少的利潤損失

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Types of Inventory 以功能區分

- Cycle Inventory: Inventory that varies between orders.
- Safety Stock Inventory: surplus inventory that protects against uncertainties in demand, lead time, and supply changes. 預測需求以外的安全庫存
- Anticipation Inventory: used to absorb uneven rates of demand or supply. 旺季或漲價前
- Pipeline Inventory 在途庫存: inventory in transit between two stocking points.

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Inventory Reduction Tactics

- Cycle inventory: Reduce the lot size
 - Reduce ordering and setup costs and allow Q to be reduced
- Safety stock inventory: Place orders closer to the time when they must be received
 - Improve demand forecasts
 - Reduce lead times and supply uncertainties
- Anticipation inventory: Match demand rate with production rates
 - Add new products with different demand cycles 季節互補
 - Provide off-season promotional campaigns
- Pipeline inventory: Reduce lead times
 - Find more responsive suppliers and select new carriers

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Economic Order Quantity

- The lot size, Q , that minimizes total annual inventory holding and ordering costs. 庫存管理總成本最低的固定訂貨量
 1. Demand rate is constant and known with certainty.
 2. The only two relevant costs are the inventory holding cost and the fixed cost per lot for ordering or setup.
 3. Decisions for one item can be made independently of decisions for other items.
 4. The lead time is constant and known with certainty.
- Use the EOQ 經濟訂貨量
 - Make-to-stock strategy with relatively stable demand.
 - Carrying and setup costs are known and relatively stable

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Calculating EOQ

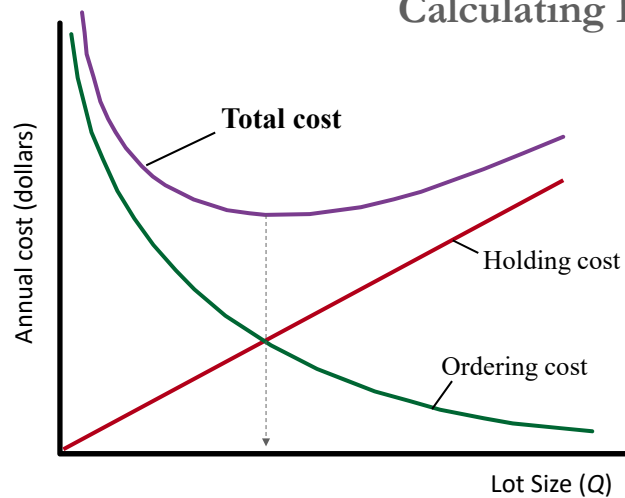
- Annual Demand = D
- Annual holding cost
= (Average cycle inventory) $\times$ (Unit holding cost) = $\frac{Q}{2} \times H$
- Annual ordering cost
= (Number of orders/Year) $\times$ (Ordering or setup costs) = $\frac{D}{Q} \times S$

$$\text{Total costs} = C(Q) = \frac{Q}{2} H \times 1 + \frac{D}{Q} S$$

$$\frac{d}{dQ} C = 0 \Rightarrow \text{EOQ} = Q^* = \sqrt{\frac{2DS}{H}} \quad \text{Time Between Orders} = \frac{\text{EOQ}}{D}$$

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Calculating EOQ



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Example 9.2: Bird Feeder

- Sales are 18 units per week, the supplier charges \$60 per unit.
- Ordering cost is \$45.
- Annual holding cost is 25 percent of a feeder's value.
- What is the annual cycle-inventory cost of the current policy of using a 390-unit lot size? Would a lot size of 468 be better?

$$D = (18 \text{ units/week})(52 \text{ weeks/year}) = 936 \text{ units}$$

$$H = 0.25(\$60/\text{unit}) = \$15 \quad S = \$45$$

$$\begin{aligned} C(Q) &= \frac{Q}{2} H + \frac{D}{Q} S = \frac{390}{2} (\$15) + \frac{936}{390} (\$45) \\ &= \$2,925 + \$108 = \$3,033 \end{aligned}$$

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Example 9.3 EOQ for bird feeders

$$EOQ = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(936)45}{15}} = \mathbf{74.94 \text{ or } 75 \text{ units}}$$

Parameters

Current Lot Size (Q)	390	Economic Order Quantity	75
Demand (D)	936		
Order Cost (S)	\$45		
Unit Holding Cost (H)	\$15		

Annual Costs

Orders per Year	2.4
Annual Ordering Cost	\$108.00
Annual Holding Cost	\$2,925.00
Annual Inventory Cost	\$3,033.00

Annual Costs based on EOQ

Orders per Year	12.48
Annual Ordering Cost	\$561.60
Annual Holding Cost	\$562.50
Annual Inventory Cost	\$1,124.10

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Inventory Control Systems

Periodic Review System



Fixed Order Period
P model

Continuous Review System



Fixed Order Quantity
Q model

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Continuous Review System 持續監控庫存

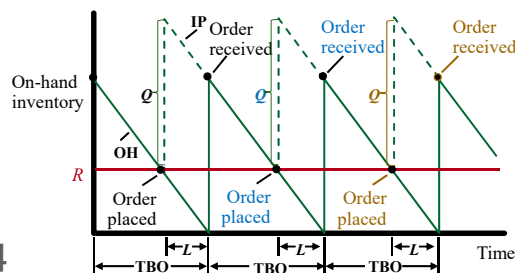
- Q model or fixed order quantity system
 - Tracks inventory position (IP)
 - Reorder point system (ROP) and fixed order quantity (Q)
 - Decision Includes scheduled receipts (SR), on-hand inventory (OH), and back orders (BO)

$$\text{Inventory position (IP)} = \text{OH} + \text{SR} - \text{BO}$$

存貨水準
現有
已訂未到

Rule: If $IP \leq ROP$, place an order of size Q

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Example 9.4

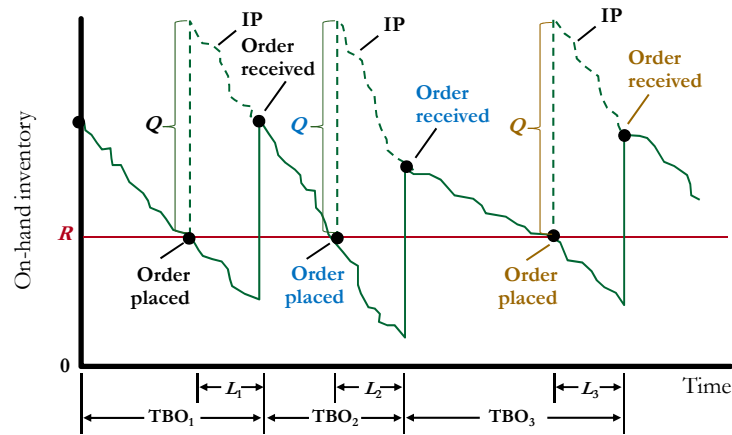
Demand for chicken soup at a supermarket is always 25 cases a day and the lead time is always 4 days. The on-hand inventory has only 10 cases. No backorders currently exist, but there is **one open order in the pipeline for 200 cases**. Should a new order be placed?

$$R = \text{Total demand during lead time} = (25)(4) = 100 \text{ cases}$$

$$IP = OH + SR - BO = 10 + 200 - 0 = 210 \text{ cases}$$

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Continuous Review System 2: Variable Demand



Select ROP base on Demand, Lead Times, and **Safety Stock**.

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Example 9.5 ROP=300, Q=250, L=5 days

Day	Demand	OH	SR	BO	IP	Q
1		400			$400 + 0 = 400$	
2	60	340			$340 + 0 = 340$	
3	80	260	250 after ordering		$260 < R$ before ordering $260 + 250 = 510$ after ordering	250 due Day 8
4	40	220	250		$220 + 250 = 470$	
5	75	145	250		$145 + 250 = 395$	
6	55	90	250		$90 + 250 = 340$	
7	95	0	$250 + 250 = 500$ after ordering	5	$0 + 250 - 5 = 245 < R$ before ordering $245 + 250 = 495$ after ordering	250 due Day 12
8	50	$250 - 50 - 5 = 195$	250		$195 + 250 = 445$	

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Continuous Review System 2: Reorder Point

- Constant lead time and variable demand 前置期間內的需求不確定

Reorder point = Average demand during lead time + Safety stock

$$= \bar{d}L + \text{Safety stock}$$

額外保留的商品量以避免缺貨

- Choosing a Reorder Point
 - Determine the distribution of demand during lead time
 - Choose an appropriate service-level policy 對商品不缺貨的要求
 - Use the formula to calculate the safety stock and reorder point levels

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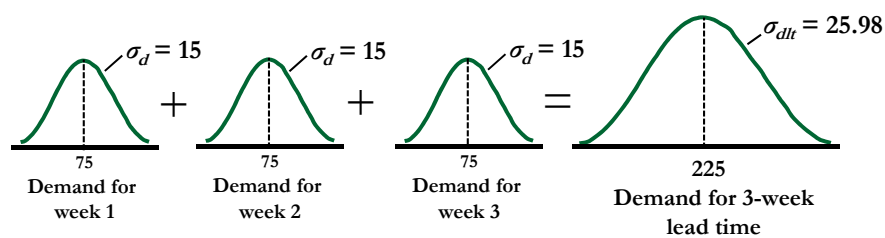
Distribution of Demand during Lead Time

- Specify mean and standard deviation of daily demand

$$\bar{d} \quad \sigma_d$$

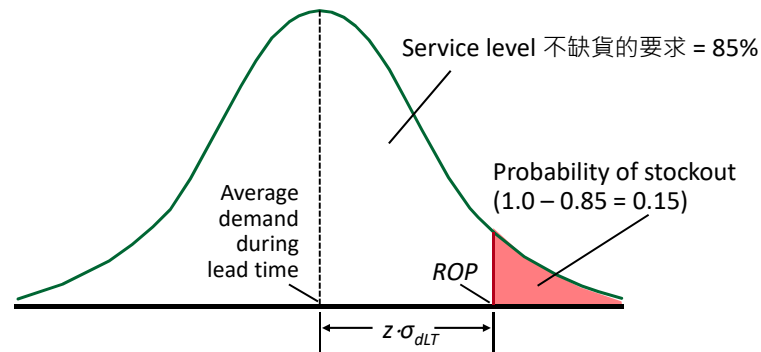
- Standard deviation of demand during lead time

$$\sigma_{dLT} = \sqrt{\sigma_d^2 L} = \sigma_d \sqrt{L}$$



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Continuous Review System 2: Service Level



$$\text{Service level} = 85\% \Rightarrow z = \Phi(0.85) = \text{NORMINV}(0.85, 0, 1) = 1.036$$

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Continuous Review System 2 : Safety Stock

$$\text{Safety stock} = z \cdot \sigma_{dLT} = z \cdot \sigma_d \sqrt{L} \quad \text{3個影響因素}$$

z = number of standard deviations needed to achieve the service level

σ_d = stand deviation of daily demand

σ_{dLT} = stand deviation of demand during lead time

If $\bar{d}=75$, $\sigma_d=15$, $L=3$ and service level=85%,
then safety stock = $1.036 \cdot 15\sqrt{3} = 26.92$

$$\text{Reorder point} = R = \bar{d}L + \text{safety stock} = (75)3 + 26.92$$

$$\text{If service level}=95\%, \text{ safety stock} = 1.645 \cdot 15\sqrt{3}$$

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Continuous Review System 3

- Both Demand and Lead Time are Variable 如果交期也不確定

$$ROP = (\text{Average daily demand} \times \text{Average lead time}) + \text{Safety stock}$$

$$= \bar{d} \bar{L} + \text{Safety stock} = \bar{d} \bar{L} + z \sigma_{dLT}$$

where

$\bar{d}$ = average daily (or weekly or monthly) demand

$\bar{L}$ = average lead time (相同時間單位)

$$\sigma_{dLT} = \sqrt{\bar{L} \sigma_d^2 + \bar{d}^2 \sigma_{LT}^2}$$

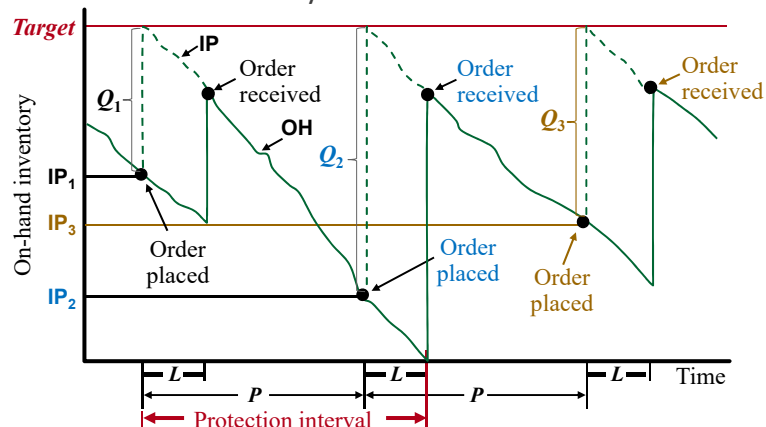
σ_d = standard deviation of daily demand

σ_{LT} = standard deviation of the lead time

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Periodic Review System 固定週期，不管存貨多寡

- Fixed interval reorder system or P model



Rule: Check inventory (IP) and then place order to meet target

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Example 9.8 Target = 620, P = 6 days, L = 5 days

Day	Demand	OH	SR	BO	IP	Q
1		400			400	
2	60	340	280 after ordering		340 before ordering 340+280=620 after ordering	620 - 340 = 280 due Day 7
3	80	260	280		260 + 280 = 540	
4	40	220	280		220 + 280 = 500	
5	75	145	280		145 + 280 = 425	
6	55	90	280		90 + 280 = 370	
7	95	90+280-95=275			275 + 0 = 275	
8	50	225	395 after ordering		225 before ordering 225+395=620 after ordering	620-225=395 due Day 13

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Periodic Review: variable demand & fixed lead time

- Selecting the time between reviews 檢查週期 $P = \frac{EOQ}{\bar{d}}$
- Set **Target Inventory Level**. 目標庫存上限 T must cover demand over a protection interval of $P + L$
 - Average demand during the protection interval is $\bar{d}(P + L)$
 - Safety stock for the protection interval = $z \cdot \sigma_{P+L} = z \cdot \sigma_d \sqrt{P + L}$
 - z is determined by the service level.
 - $T = \bar{d}(P + L) + z \cdot \sigma_d \sqrt{P + L}$
- **Order Quantity Q** = T – Inventory on hand (OH)

當期訂購量 目標庫存上限 現有庫存

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Example 9.9: Bird Feeder

- Weekly demand for the bird feeder is normally distributed with a mean of 18 units and a standard deviation of 5 units.
- The lead time is 2 weeks. The Q system called for an EOQ of 75 units and a safety stock of 9 units for a cycle-service level of 90%.
- What is the equivalent P system?

$$P = \frac{EOQ}{\bar{d}} = \frac{75}{18} \approx 4 \text{ weeks} \quad \text{Service level of 90\%} \Rightarrow z=1.28$$

$$\begin{aligned} T &= \bar{d}(P + L) + z \cdot \sigma_d \sqrt{P + L} = 18(4+2) + 1.28(5)\sqrt{4+2} \\ &= 108 + 15.68 \approx 124 \end{aligned}$$

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Costs for Q and P Systems

- Total Annual Q System Costs

$$TC = \frac{Q}{2}(H) + \frac{D}{Q}(S) + (\text{Safety stock})(H)$$

- Total Annual P System Costs

$$= \frac{\bar{d}P}{2}H + \frac{D}{\bar{d}P}S + (\text{safety stock})H$$

- The P system requires more inventory for the same level of protection against stockouts or backorders.
- **Optional Replenishment System** 選擇補貨系統

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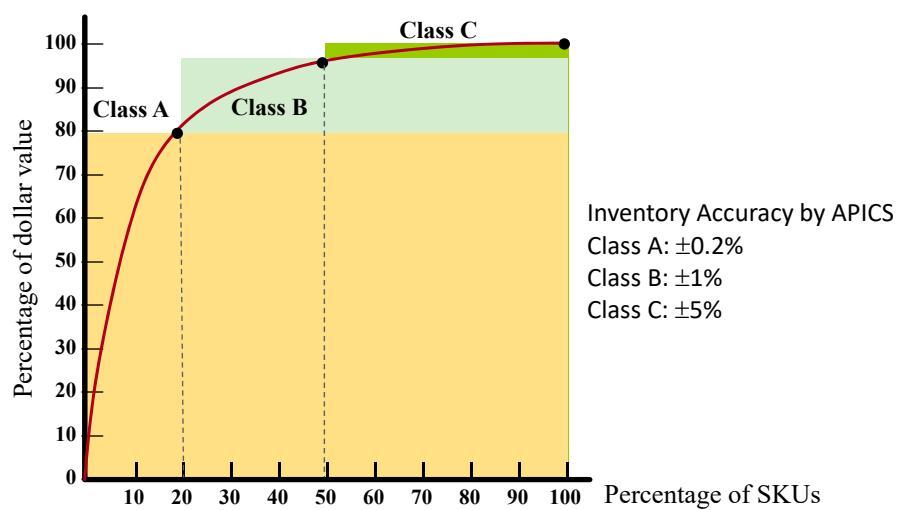
ABC庫存分類管理 Pareto's 80/20 principle

SKU no.	annual demand	unit cost	annual dollar value
1	2500	330	825000
2	1000	70	70000
3	1900	500	950000
4	1500	100	150000
5	3900	700	2730000
6	1000	915	915000
7	200	210	42000
8	1000	4000	4000000
9	8000	10	80000
10	9000	2	18000
11	500	200	100000
12	400	300	120000

There are other ways to do ABC classification. Review ABC classification periodically.

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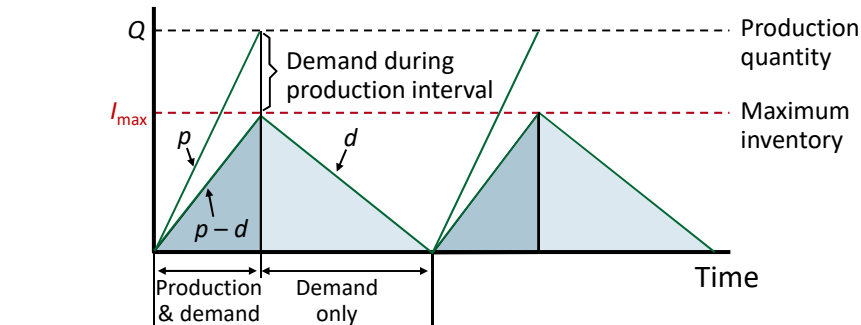
Pareto Chart for ABC Analysis



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Supplement C: Economic Production Quantity

- 經濟生產批量 Production Quantity Q .
- Production rate $p >$ the demand rate d , so there is a buildup of $(p - d)$ units per time period.
- Buildup continues for Q/p periods.



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Economic Production Quantity economic lot size

Max. inventory is: $I_{max} = (p - d) \times \text{production time} = (p - d) \frac{Q}{p}$

Average inventory is no longer $Q/2$, it is $I_{max}/2$

Total annual cost = Annual holding cost + Annual setup cost

$$C = \frac{I_{max}}{2}(H) + \frac{D}{Q}(S) = \frac{Q}{2} \left(\frac{p-d}{p} \right) (H) + \frac{D}{Q}(S)$$

$$\text{成本最小化} \Rightarrow EPQ = ELS = \sqrt{\frac{2DS}{H}} \sqrt{\frac{p}{p-d}}$$

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Example C.1

A plant manager must decide the lot size for a product that has a steady demand of **30 units per day**. The production rate is **190 units per day**, annual demand is **10,500 units**, setup cost is **\$200**, annual holding cost is **\$0.21 per units**, and the plant operates **350 days per year**.

$$EPQ = \sqrt{\frac{2DS}{H}} \sqrt{\frac{p}{p-u}} = \sqrt{\frac{2(10500)(200)}{0.21}} \sqrt{\frac{190}{190-30}} = 4873.4$$

$$C = \frac{Q}{2} \left(\frac{p-d}{p} \right) (H) + \frac{D}{Q} (S) = \frac{4873.4}{2} \left(\frac{190-30}{190} \right) (0.21) + \frac{10500}{4873.4} (200)$$

Q: What are the advantages of reducing the setup time by 10%?

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Supplement C: One-Period Decisions

- 適用於不可庫存或庫存價值低之商品採購與製造、旅遊業超額訂位
- Example C.3: A gift shop sells a Christmas ornament carved from wood and makes a **\$10 profit per unit** sold during the season, but it takes a **\$5 loss per unit** after the season is over.

Step 1: List the demand levels and probabilities.

Demand	10	20	30	40	50
Demand Probability	0.2	0.3	0.3	0.1	0.1

Step 2: Develop a payoff table that shows the profit for each purchase quantity, Q , at each assumed demand level, D .

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One-Period Decisions

Payoff = (Profit per unit)(Demand quantity) – (Loss per unit)(Leftovers)

$$= \begin{cases} p \cdot Q & \text{if Demand} \geq \text{Order Quantity} \\ p \cdot D - l(Q - D) & \text{if Demand} < \text{Order Quantity} \end{cases}$$

		Demand					Expected Payoff
		10 (0.2)	20 (0.3)	30 (0.3)	40 (0.1)	50 (0.1)	
Order Quantity	10	100	100	100	100	100	100
	20	50	200	200	200	200	170
	30	0	150	300	300	300	195
	40	-50	100	220	400	400	175
	50	-100	50	200	350	500	140

Steps 3 & 4: Calculate the expected payoff of each Q and pick the best.

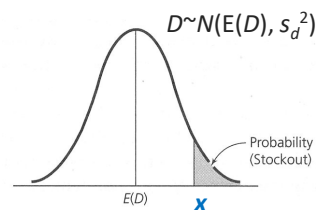
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Newsboy Problem

預測報紙需求 D 為常態分布

平均值 = 90 papers 標準差 = 10 papers

進貨 90 papers $\Rightarrow P(\text{stockout}) = 50\%$



C_e = 進貨高估需求的單位成本 = 進價 - 殘值 報紙進價 = 0.20

C_s = 進貨低估需求的單位成本 = 售價 - 進價 報紙利潤 = 0.30

目前進貨 90 份，增加進貨是否能增加利潤？

$$P = P(\text{增加的進貨賣不出}) = 0.5 \Rightarrow (1-P) \cdot C_s = 0.5(0.30) > P \cdot C_e = 0.5(0.20)$$

潛在利潤 潛在損失

Key: 增加進貨量直到 $P \approx \frac{C_s}{C_e + C_s} \leftarrow \text{critical ratio or optimal service level}$

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Airline Overbooking (Revenue Management)

飛機有90個座位，經常有人訂位卻臨時取消或延期

假設 probability of no-show=10%，訂位額滿時預期會有9個空位

Solution: 超額訂位

Decision: 接受100個訂位 $\Rightarrow$ no-shows $\sim N(10, s^2)$

$\Rightarrow P(\text{no-shows} < 10) = 50\% \Rightarrow P(\text{overbooking}) = 50\%$

訂位系統高估 no-show (座位不足) 的單位成本 $= C_e$

低估 no-show (旅客未報到) 的單位成本 $= C_s$

$P(\text{no overbooking}) > 80\% \Rightarrow$ 減少超額訂位

Q: 如果 $C_s \ll C_e$ ，航空公司應如何調整超額訂位？

