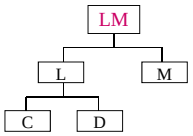
Chapter 13 Inventory Management

- Inventory Costs
- EOQ and EPQ models
- Q models and P models
- Single-Period Model

庫存的分類

Independent Demand ready to be sold or used, not affected by other products

Dependent Demand
components of finished products



Manufacturing Inventory
raw materials, replacement parts, supplies, finished goods
Dependent: raw materials, component parts, WIP

Retail Inventory
tangible goods to be sold, supplies, goods in transit

Purposes of Inventory

To meet anticipated demand (FGI)

To smooth production requirements (FGI)

To decouple operations (WIP)

To protect against stockouts (materials, components)

To take advantage of order cycles

To help hedge against price increases

To permit operations (WIP)

To take advantage of quantity discounts

4

Inventory Costs

- Holding or Carrying cost ← order too much or too early
  storage cost 倉儲設備、進出盤點
  risk cost 貶值、偷竊、毀損、保費
  opportunity cost 資金積壓的潛在損失
- Ordering or Setup cost 前置作業成本
  對外採購：聯繫、運輸、驗收　內部製造：停機調整、試產
- Shortage costs or Lost Sales ← order too little or too late
  停工減產的損失、延誤交貨的罰款
  銷售減少的利潤損失
- Annual cost≈20% to 40% of the inventory's worth

5

How Much to Order? Economic Order Quantity

Assumptions:

Only one product is involved

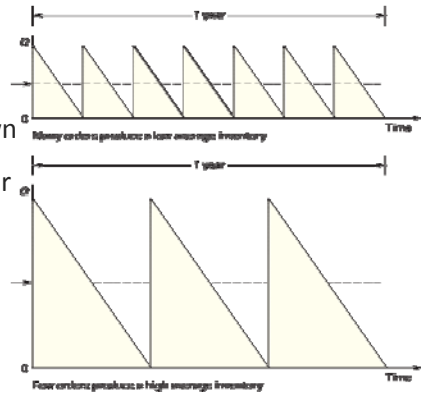
Annual demand requirements known

Demand is even throughout the year

Lead time does not vary

Each order is received in a single delivery

There are no quantity discounts



Economic Order Quantity

D=全年需求預測 S=單次訂貨成本 H=單位持有成本 Q=訂貨量

單位:$/unit/year

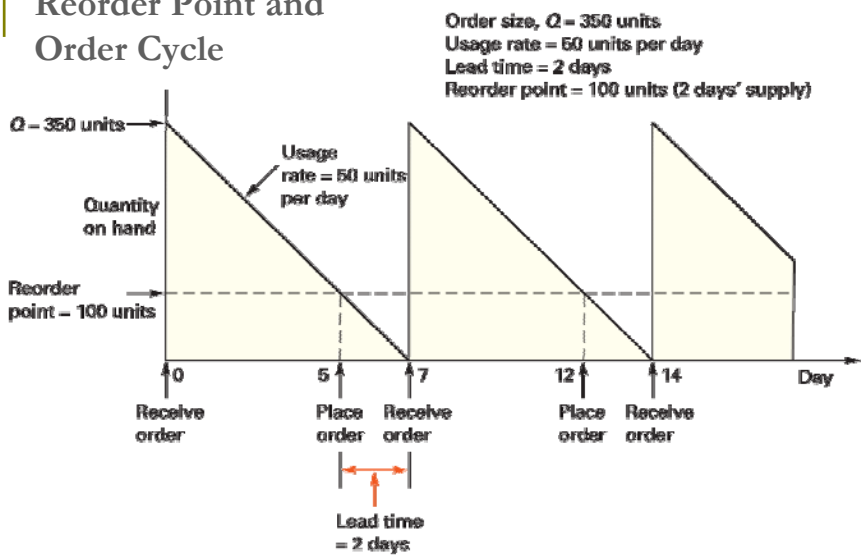
訂貨次數 $=\frac{D}{Q}$，平均庫存 $=\frac{Q}{2}$

$TC(Q)=\frac{D}{Q}S+\frac{Q}{2}H\times 1$ 全年庫存成本=訂貨成本+持有成本

$\frac{d}{dQ}TC=0\Rightarrow Q^{*}=\sqrt{\frac{2DS}{H}}$ Length of order cycle $=\frac{Q^{*}}{D}$

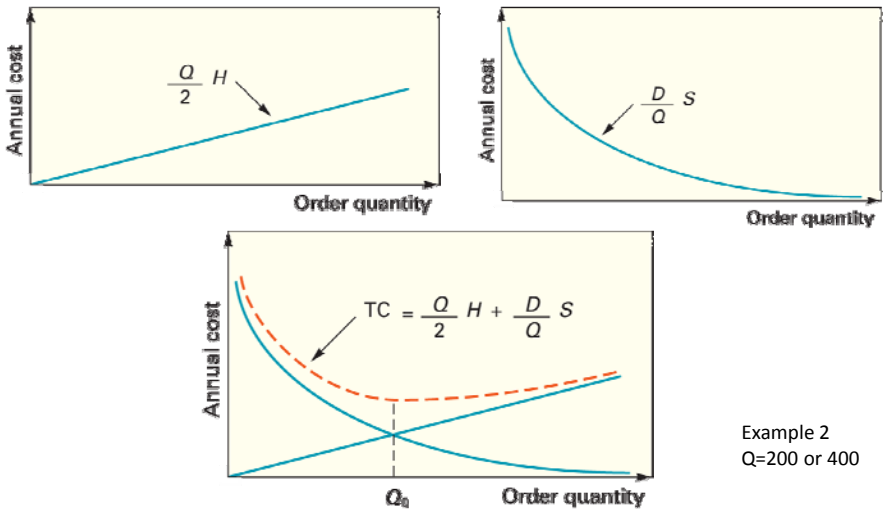
Example 2

Reorder Point and Order Cycle



8

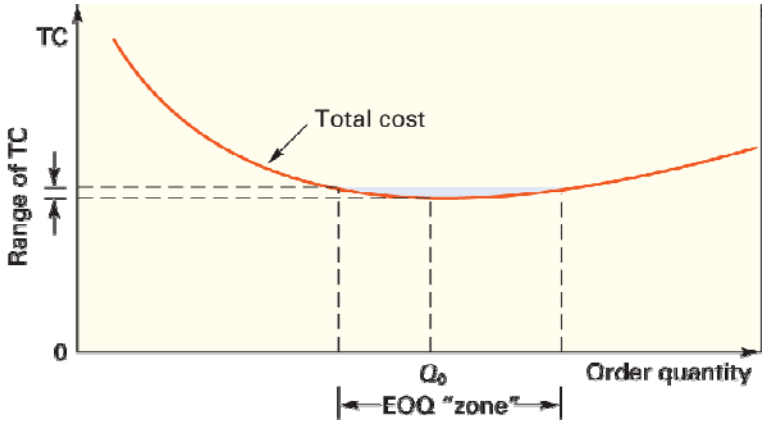
Carrying Cost, Ordering Cost, and Total Cost



Example 2
Q=200 or 400

9

EOQ is an Approximate Quantity



10

EOQ with Quantity Discounts

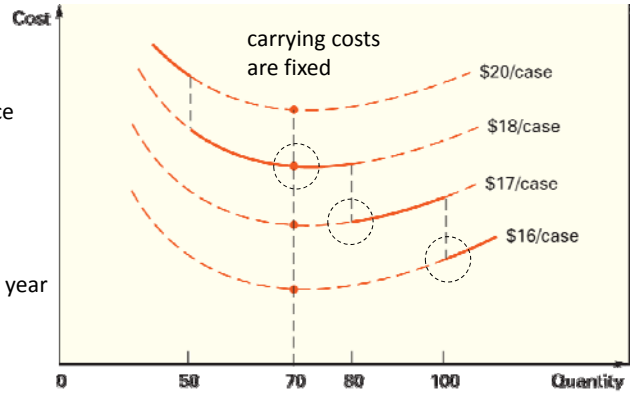
產品購買價格 P
因採購量多寡而不同

$$TC(Q) = PD + \frac{D}{Q}S + \frac{Q}{2}H$$

Example 5

Range	Unit Price
1 to 49	$20
50 to 79	$18
80 to 99	$17
≥100	$16

H = $4 per case per year



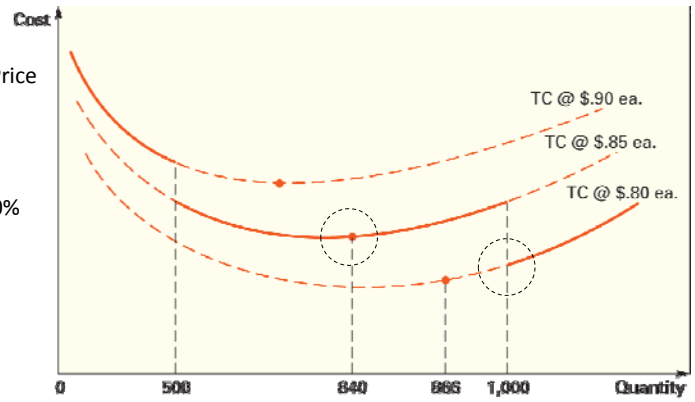
11

Carrying Costs as a Percentage of Purchasing Price

Example 6

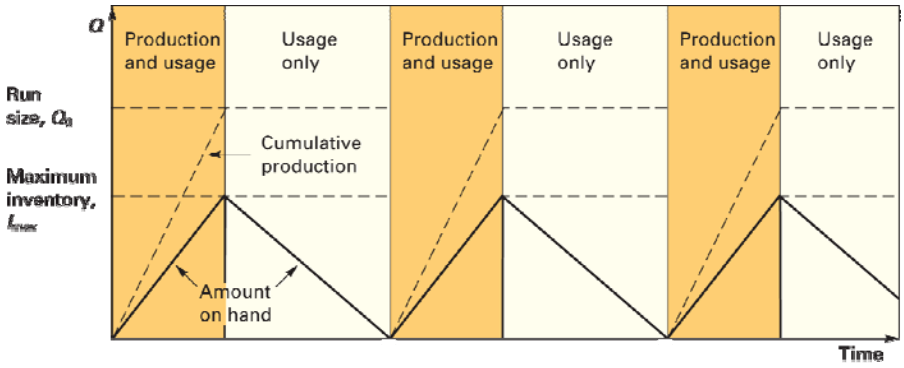
Range	Unit Price
1 to 499	$0.90
500 to 999	$0.85
≥1000	$0.80

Holding cost = 40%



12

Economic Production Quantity: EOQ for Production



$$EPQ = \sqrt{\frac{2DS}{H}}\sqrt{\frac{p}{p-u}} \qquad I_{\max} = \text{run time} \times (p-u) = \frac{EPQ}{p}(p-u)$$

Example 4

13

Inventory Ordering Policies

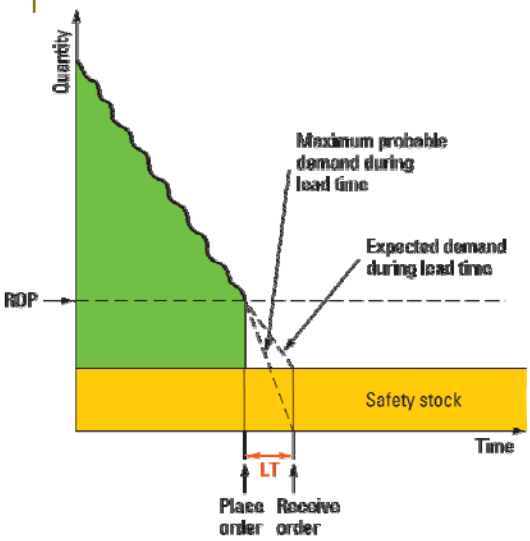
Question: How to order and control inventory?

Fixed Order Period
P model

Fixed Order Quantity
Q model

14

When to Order: 固定訂貨量模式 Q model



當庫存降至訂貨點ROP時訂貨，訂貨量Q固定不變

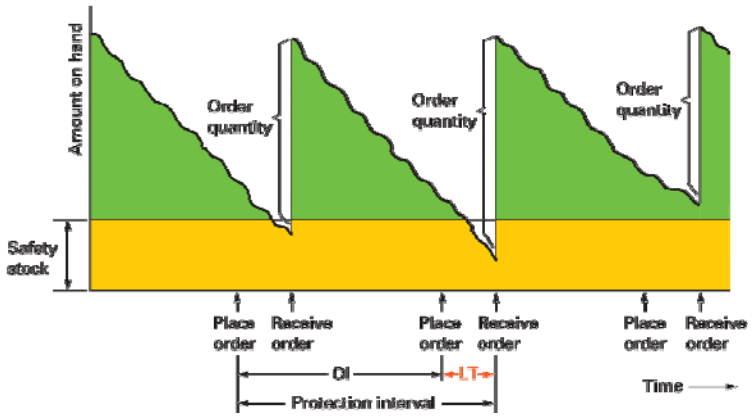
Decision: ROP=?

適用於較重要商品的補貨，Make to Stock的生產方式

15

How Much to Order: 固定訂貨週期模式 P model

適用於一般性商品的採購

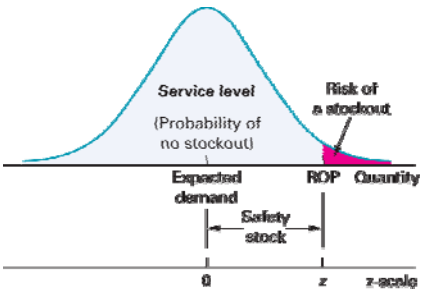


每隔一段固定時間盤點庫存，根據庫存結餘決定訂貨量

16

Safety Stock

- 庫存超過預測需求的數量，以避免需求增加而造成缺貨損失
- depends on service level, average demand, demand variability, order lead time
- service level depends on Holding cost ⇔ Shortage cost



Decision: 庫存準備量等於預測需求

⇒ P(stockout)=P(D>E(D))=50%

Service level=P(no stock out)>80%

⇒ $P(D<E(D)+z\cdot s_{dLT})>80\%$

safety stock

17

Fixed Order Quantity with Safety Stock

考量時間範圍=lead time (*LT*)

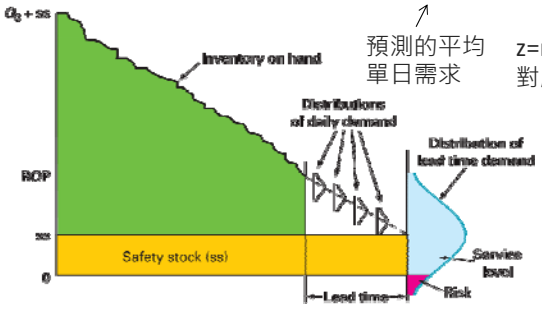
If only demand is variable, find standard deviation of demand σ_d

所需庫存量= $ROP = \text{expected demand} + \text{safety stock}$

$= \bar{d} \times LT + z\sigma_d\sqrt{LT}$

預測的平均單日需求

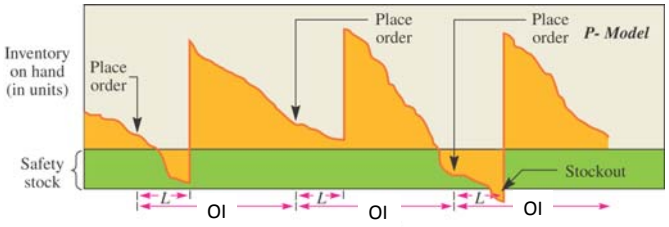
z=number of standard deviations 對應至service level (不缺貨機率)



Example 9

Fixed Order Interval with Safety Stock

Decision
訂貨量Q=?



考量時間範圍= *OI* + *L*=固定週期天數+交貨天數

所需庫存量= Expected demand + Safety stock = 現有庫存量*A* + 訂貨量Q

⇒ Q = Expected demand + Safety stock - 現有庫存量*A*

$= \bar{d}(OI + LT) + z\sigma_d\sqrt{OI + LT} - A$

Example 13

Objectives of Inventory Control

Average Inventory=$\frac{Q}{2}+SS$ $\frac{d\cdot \mathrm{OI}}{2}+SS$

service level= $\frac{\text{庫存用完的次數}}{\text{訂貨次數}}$ fill rate = $\frac{\text{需求立即被滿足的數量}}{\text{總需求數量}}$

- Annual Inventory turn = $\frac{\text{annual cost (\$) of goods sold}}{\text{average inventory value (\$)}}$
- Weeks of supply = $\frac{\text{average inventory}}{\text{average weekly demand}}$ Days of supply

Effective Inventory Management

- A system to keep track of inventory on hand and on order.
  periodic counting, perpetual counting.
- A reliable forecast of demand.
- Knowledge of lead times and lead time variability.
- Reasonable estimates of inventory costs.
  holding costs, ordering costs, shortage costs.
- A classification system for inventory items.

Hospitals and Inventory Management

Lahey CLINIC

Control

- Barcodes and computers keep track of every bottle of antibiotics and other supplies.
- Secure supply cabinets with thumbprint security technology

Management

- Analyze how much is spent on every type of illness and surgical procedure.
- Computers keep track of stock and automatically reorder from suppliers

22

ABC庫存分類管理

Pareto's 80/20 principle
義大利80%的財富為20%的人所掌握

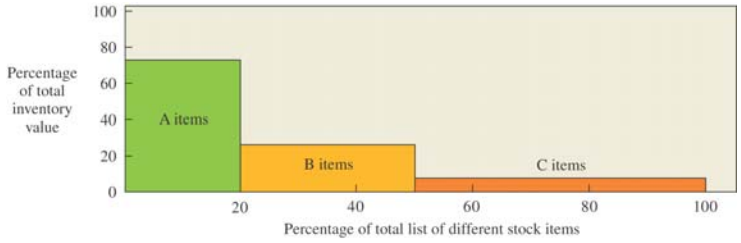
Example 1

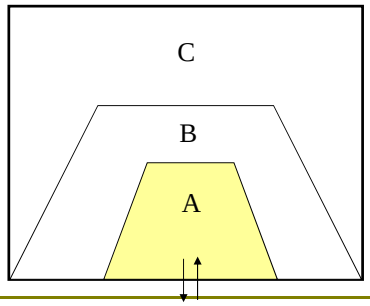
item no.	annual demand	unit cost	annual dollar value
1	2500	330	**825000**
2	1000	70	70000
3	1900	500	**950000**
4	1500	100	150000
5	3900	700	**2730000**
6	1000	915	**915000**
7	200	210	42000
8	1000	4000	**4000000**
9	8000	10	80000
10	9000	2	18000
11	500	200	100000
12	400	300	120000

There are other ways to do ABC classification. Review ABC classification periodically.

23

ABC分類於庫存控管與儲位配置的應用





包裝食品與日用品
（以紙箱或散裝為出貨單位）

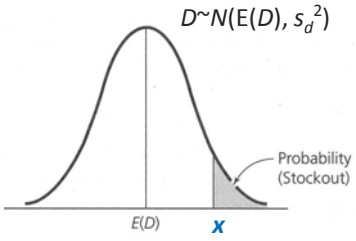
飲料（以棧板為出貨單位）

出貨區

Single Period Inventory Control

適用於庫存價值低之流行商品採購與製造、旅遊業超額訂位

1. 預測報紙需求D為常態分布
平均值=90 papers　標準差=10 papers



Decision: 進貨90 papers
$\Rightarrow P(\text{stockout})=P(D>90)=P(D>E(D))=50\%$

Goal: $P(\text{stockout})<20\%$
$\Rightarrow P(\text{no stockout})>80\% \Rightarrow P(D<x)>80\% \Rightarrow P(D<E(D)+z\cdot s_d)>80\%$

Decision: 進貨90+0.84162(10)=99 papers
無條件進位

Newsboy Problem

C_e=進貨高估需求的單位成本=進價-殘值　　報紙進價=0.20

C_s=進貨低估需求的單位成本=售價-進價　　報紙利潤=0.30

預測報紙需求為90份，目前進貨90份，增加進貨是否能增加利潤？

P=P(增加的進貨賣不出)=P(需求<90)=0.5

⇒ 潛在利潤 $(1-P)\cdot C_s$=0.5(0.30) > 潛在損失 $P\cdot C_e$=0.5(0.20)

Key: 增加進貨量直到 $P \approx \frac{C_s}{C_e + C_s}$ ← Optimal service level

Q: 如果 $C_s << C_e$，我們應如何調整進貨量？

Airline Overbooking (Yield Management)

2. 飛機有90個座位，經常有人訂位卻不到

假設 probability of no-show=10%，訂位額滿時預期會有9個空位

Solution: 超額訂位

Decision: 接受100個訂位 ⇒ no-shows ~$N(10, s^2)$

⇒ P(no-shows<10)=50% ⇒ P(overbooking)=50%

Goal: P(no overbooking)>80% ⇒ 減少超額訂位

高估 no-show 人數(座位不足)的單位成本=C_e

低估 no-show 人數(座位未售)的單位成本=C_s

Conclusion

- Inventory Management要能支持營運，但要避免過高的庫存成本
- 庫存品項眾多時，應分類管理並保持準確的庫存紀錄
- 庫存管理要能有效預測需求變異
- Order size與safety stock的決策需考慮庫存成本與服務水準的平衡
- 趨勢是與供應商或客戶合作，同時降低成本與缺貨率

28