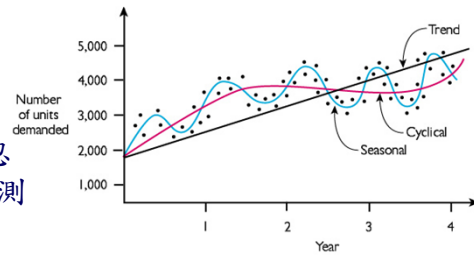


Trend, Seasonality, Cycles

- Seasonality是預測對象在各個週期都重複出現的變化型態
- 週期：一年、一月、一週
- 產品銷售量的seasonality通常是因氣候變化、風俗習慣、社會制度等因素而產生
- 銷售通常同時受到trend, seasonality, cycle的影響
- 評量長期發展趨勢時，可忽略seasonality變化，細部預測則必須納入所有因素影響



1

Seasonality

- 週期：年(4,12)、月(30)、週(7)
- Simple Average Method

	week1	week2	week3	average	seasonal factor
Tues	67	60	64	63.67	0.8896
Wed	75	73	76	74.67	1.0432
Thur	82	85	87	84.67	1.1830
Fri	98	99	96	97.67	1.3646
Sat	90	86	88	88.00	1.2295
Sun	36	40	44	40.00	0.5589
Mon	55	52	50	52.33	0.7312

Overall Average = 71.57

2

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Centered Moving Averages

以當期為基準，將前後期併入平均值計算，以抵銷季節性的影響

For weekly data:

$$CMA_t = (Y_{t-3} + Y_{t-2} + Y_{t-1} + Y_t + Y_{t+1} + Y_{t+2} + Y_{t+3}) / 7$$

For monthly data:

$$MA_t = (Y_{t-6} + Y_{t-5} + \dots + Y_t + Y_{t+1} + \dots + Y_{t+5}) / 12$$

Need adjustment if the number of periods is even.

$$CMA_t = (MA_t + MA_{t+1}) / 2$$

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Seasonal Index Based on CMA

Period	Season	Actual	Centered MA	Seasonal Factor	
1	Tues	67			
2	Wed	75			
3	Thur	82			
4	Fri	98	71.857	1.3638	
5	Sat	90	70.857	1.2702	
6	Sun	36	70.571	0.5101	
7	Mon	55	71.000	0.7746	
8	Tues	60	71.143	0.8434	
9	Wed	73	70.571	1.0344	
10	Thur	85	71.143	1.1948	
11	Fri	99	70.714	1.4000	
12	Sat	86	71.286	1.2064	1.2383
13	Sun	40	71.714	0.5578	0.5339
14	Mon	52	72.000	0.7222	0.7484
15	Tues	64	71.571	0.8942	0.8688
16	Wed	76	71.857	1.0577	1.0460
17	Thur	87	72.429	1.2012	1.1980
18	Fri	96	72.143	1.3307	1.3648
19	Sat	88			
20	Sun	44			0.9998
21	Mon	50			

$$98 / 71.857 = 1.3638$$

$$\begin{aligned} \text{seasonal index} \\ &= 1.2383 / 0.9998 \\ &= 1.2386 \end{aligned}$$

0.5341
0.7486
0.8690
1.0463
1.1983
1.3652

4

I. Forecasting Trend and Seasonality

Preparation: 由資料變化型態主觀判斷是否同時有trend與seasonality的影響。

Approach 1: 運用可直接處理seasonality的預測方法

Multiple Regression, Holt-Winters Smoothing

Approach 2: Time Series Decomposition

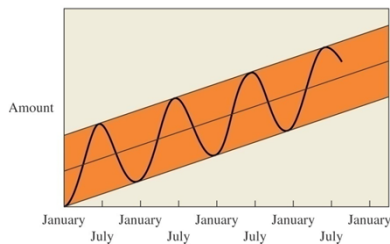
1. 根據數個週期的數據估算seasonal factors。
2. 將原始數據deseasonalized。
3. 以迴歸分析deseasonalized數據的趨勢並進行預測。
4. 將預測再乘上seasonal factors的影響

Trend and Seasonal Factors

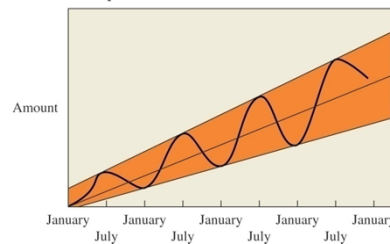
Additive: trend+seasonal

Multiplicative: trend×seasonal

A. Additive Seasonal



B. Multiplicative Seasonal



次年預測1100

第1季	第2季	第3季	第4季	合計
200	350	300	150	1000
200/250	350/250	300/250	150/250	

第1季	$275 \times 0.8 = 220$
第2季	$275 \times 1.4 = 385$
第3季	$275 \times 1.2 = 330$
第4季	$275 \times 0.6 = 165$

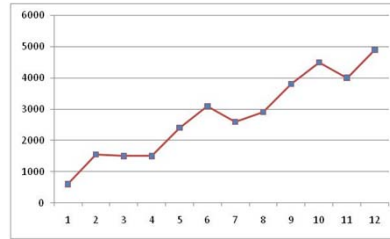
Decomposition Using Least Square Regression

1. Decomposition

- Calculate the seasonal factor.

$$SF_1 = \frac{(y_1 + y_5 + y_9)/3}{\bar{y}} \approx 0.816$$

$$SF_2=1.098, SF_3=0.972, SF_4=1.115$$



- Deseasonalize the demand. $y'_i = y_i / SF_k$

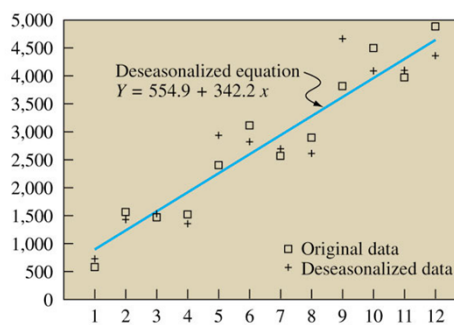
Quarter (X)	1	2	3	4	5	6	7	8	9	10	11	12
Sales (Y)	600	1550	1500	1500	2400	3100	2600	2900	3800	4500	4000	4900
Deseasonalized	736	1412	1544	1345	2943	2825	2676	2600	4659	4100	4117	4393

7

- Develop a regression model from the deseasonalized data.

$$Y' = a + bX$$

$$= 554.9 + 342.2x$$



2. Forecast

- Project the future using the regression model.

- Multiply the seasonal factor.

$$Y' = 554.9 + 342.2 \times 13 = 5003.5 \Rightarrow Y = Y' \times 0.816 = 4102.87$$

8

Decomposition Using Centered Moving Averages

For quarterly data:

$$MA_t = (Y_{t-2} + Y_{t-1} + Y_t + Y_{t+1}) / 4$$

← 抵銷季節性的影響

For monthly data:

$$MA_t = (Y_{t-6} + Y_{t-5} + \dots + Y_t + Y_{t+1} + \dots + Y_{t+5}) / 12$$

Need adjustment if the number of periods is even.

$$CMA_t = (MA_t + MA_{t+1}) / 2$$

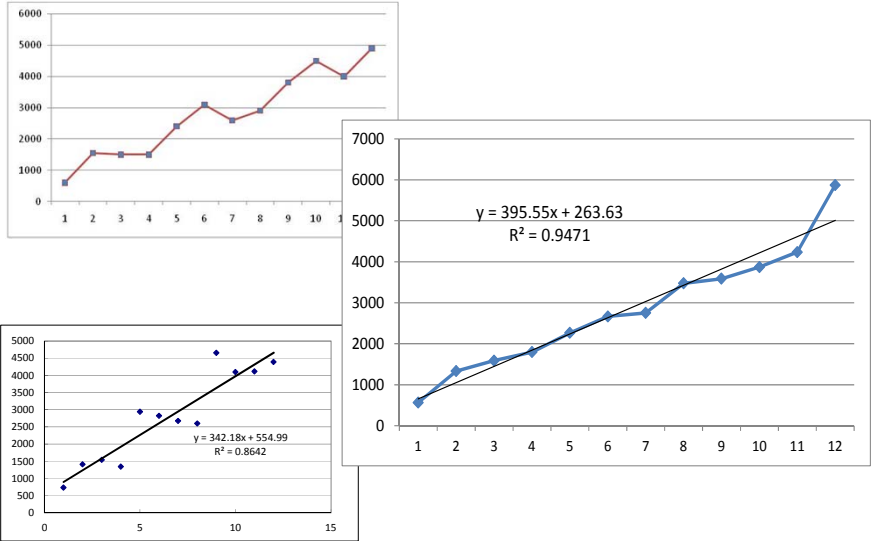
A Different Viewpoint of Decomposition

取平均值或中位數

X	Y	Moving Average	CMA	Y/CMA	Seasonal Index	Adjusted	Deseasonalized
1	600						566.62
2	1550						1333.55
3	1500	1287.5	1512.5	0.9917	0.9403	0.9442	1588.61
4	1500	1737.5	1931.3	0.7767	0.8311	0.8346	1797.37
5	2400	2125	2262.5	1.0608	1.0545	1.0589	2266.48
6	3100	2400	2575	1.2039	1.1575	1.1623	2667.10
7	2600	2750	2925	0.8889			2753.59
8	2900	3100	3275	0.8855			3474.91
9	3800	3450	3625	1.0483			3588.59
10	4500	3800	4050	1.1111			3871.59
11	4000	4300					4236.29
12	4900						5871.40
					0.9959	1	

調整季節因素使平均值為1

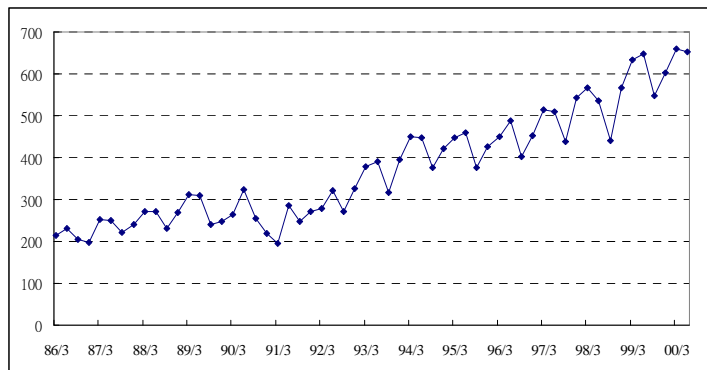
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Example: Light Truck Sales



March 1986 – June 2000

銷售具有季節性與趨勢的變化，每年第二季與第三季銷售最強勁

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Deseasonalize the Demand

seasonal factor=計算各年分同一季SF的中位值，除以整體平均值

F61 =MEDIAN(F4:F57)										
	A	B	C	D	E	F	G	H	I	J
1	Date	Trucks	Moving	CMA	SF					Deseasonalized
2	Mar-86	213.831								198.904
3	Jun-86	231.682								216.186
4	Sep-86	205.904	212.309	217.135	0.948	0.948				234.003
5	Dec-86	197.817	221.962	224.129	0.883		0.883			203.232
6	Mar-87	252.445	226.295	228.104	1.107			1.107		234.822
7	Jun-87	249.015	229.913	235.166	1.059				1.05889	232.360
8	Sep-87	220.373	240.420	242.742	0.908	0.908				250.447
9	Dec-87	239.846	245.065	247.928	0.967		0.967			246.411
10	Mar-88	271.026	250.792	252.207	1.075			1.075		252.106
11	Jun-88	271.922	253.623	257.296	1.057				1.05685	253.735
12	Sep-88	231.696	260.969	265.982	0.871	0.871				263.315
13	Dec-88	269.232	270.995	275.723	0.976		0.976			276.602
14	Mar-89	311.13	280.450	281.605	1.105			1.105		289.411
15	Jun-89	309.742	282.760	280.169	1.106				1.10556	289.025
16	Sep-89	240.937	277.577	271.737	0.887	0.887				273.817
59	Jun-00	653.024								609.347
60										
61	Average	375.1713			Median	0.885	0.979	1.082	1.078	1.006
62					Adjusted	0.880	0.973	1.075	1.072	

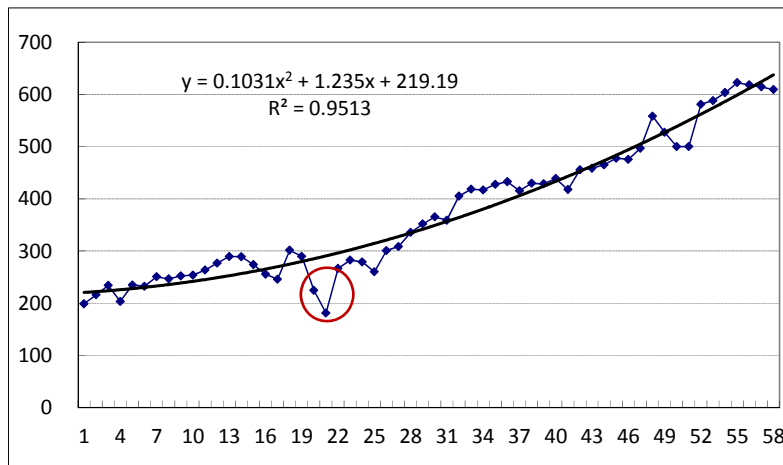
March 86: 213.831/1.075=198.904

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Estimating Trend of Deseasonalized Data

2次曲線的趨勢 $Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \epsilon$



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Forecasting Future Sales (Sep, 2000)

- Winters' Triple Exponential Smoothing
 $F=630.37, T=9.56, S=0.92 \Rightarrow \text{Forecast}=(F+T)\times S=590.64$
- Decomposition
 $x=59 \Rightarrow y = 0.1031x^2 + 1.235x + 219.19 = 652.27$
 $SF_3=0.88 \Rightarrow \text{Forecast}=572.83$

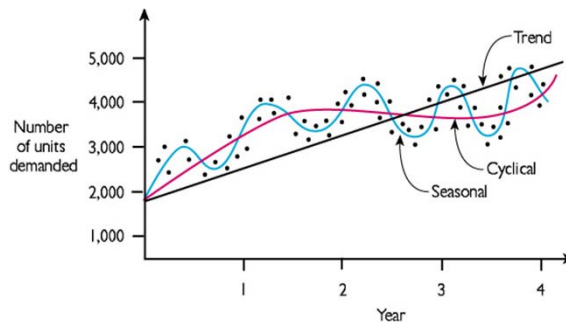
Decomposition Using Minitab

The screenshot shows the Minitab interface. The 'Stat' menu is open, and the path 'Stat > Time Series > Decomposition...' is highlighted. Below the menu, the 'Decomposition' dialog box is open, showing 'Trucks' as the variable and '4' as the seasonal length. The 'Model Type' is set to 'Multiplicative' and 'Trend plus seasonal'. The 'Storage...' button is circled in red. To the right, the 'Decomposition - Storage' dialog box is open, showing options for 'Storage' such as 'Trend line', 'Detrended data', 'Seasonals', 'Seasonally adjusted data', 'Fits', 'Residuals', and 'Forecasts'. The 'Fits' option is checked.

Note: Minitab的分解法
只能以線性趨勢分析

II. Modeling Cycles

- 銷售資料可能出現無法用趨勢或季節因素解釋的變化。
- Cycles是指經濟景氣對銷售的影響，沒有固定週期，沒有固定的幅度，影響遍及各種產品。
- Cycles的轉折點是預測的難題。



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Time-Series Decomposition Model

假設銷售量受到下列因素影響

$$Y = T \times S \times C \times I$$

Y: demand to be forecast

T: long term trend

S: seasonal indices

C: cyclical factor

I: irregularities or random variations (無法預測)

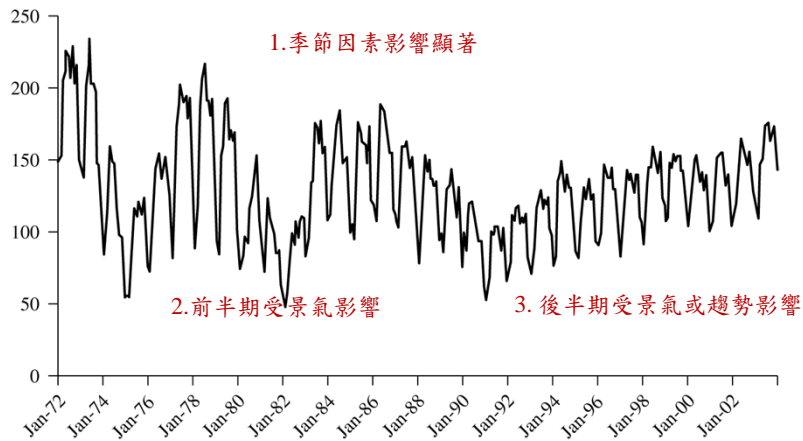
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FIGURE 6.1 Private Housing Starts in Thousands of Units by Month, 1972–2003

(c6t1&f1)

This plot of private housing starts shows the volatility in the data. There are repeated sharp upward and downward movements that appear regular and may be of a seasonal nature. There also appears to be some wavelike cyclical pattern and perhaps a slight negative trend.



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Centered Moving Averages

Step 1: CMA removes short term fluctuations.

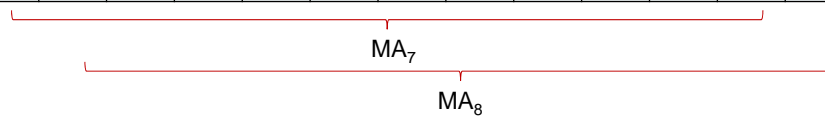
For monthly data:

$$MA_t = (Y_{t-6} + Y_{t-5} + \dots + Y_t + Y_{t+1} + \dots + Y_{t+5}) / 12$$

Need adjustment if the number of periods is even.

$$CMA_t = (MA_t + MA_{t+1}) / 2$$

Jan-72	Feb-72	Mar-72	Apr-72	May-72	Jun-72	Jul-72	Aug-72	Sep-72	Oct-72	Nov-72	Dec-72	Jan-73
149.1	152.2	203.9	211.6	225.8	223.1	206.5	228.6	203.0	216.5	185.7	150.5	146.6



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TABLE 6.2 Time-Series Decomposition of Private Housing Starts

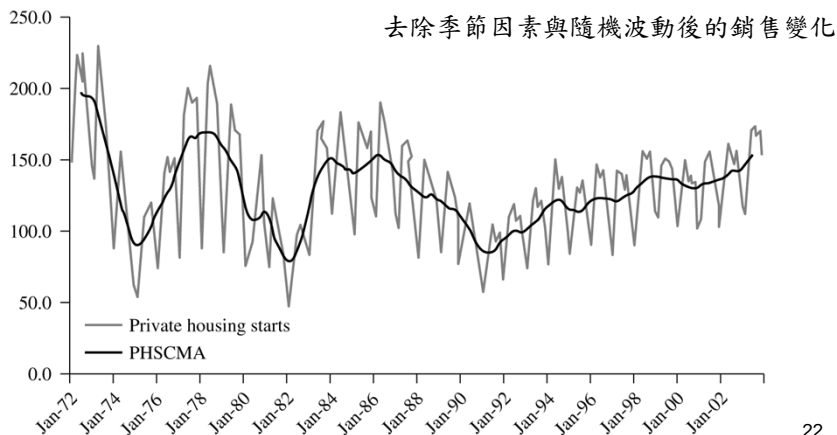
1	2	3	4	5
Period	Time Index	PHS	PHSMA	PHSCMA
Jan-72	1	149.1		
Feb-72	2	152.2		
Mar-72	3	203.9		
Apr-72	4	211.6		
May-72	5	225.8		
Jun-72	6	223.1		
Jul-72	7	206.5		
Aug-72	8	228.6	196.38	196.27
Sep-72	9	203.0	196.17	195.58
Oct-72	10	216.5	194.98	194.82
Nov-72	11	185.7	194.66	194.38
Dec-72	12	150.5	194.11	194.45
Jan-73	13	146.6	194.79	193.94
Feb-73	14	138.0	193.08	192.92
Mar-73	15	200.0	192.76	191.45
Apr-73	16	205.0	190.14	187.87
May-73	17	234.0	185.59	182.70
Jun-73	18	202.6	179.81	177.63
Jul-73	19	202.6	175.44	172.94
Aug-73	20	197.2	170.43	167.85
Sep-73	21	148.4	165.26	164.07
Oct-73	22	147.1	162.88	159.74
Nov-73	23	133.3	156.61	154.71
Dec-73	24	90.4	152.82	149.28
.	.	.	145.73	143.44
.

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FIGURE 6.2 Private Housing Starts (PHS) with the Centered Moving Average of Private Housing Starts (PHSCMA) in Thousands of Units (c6f2)

The centered moving-average series, shown by the darker line, is much smoother than the original series of private housing starts data (lighter line) because the seasonal pattern and the irregular or random fluctuations in the data are removed by the process of calculating the centered moving averages.



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Estimating Seasonal Indices

實際銷售 抵銷季節性影響後的銷售
 ↓ ↙

Step 2: Seasonal Factor $SF_t = Y_t / CMA_t$

Seasonal Index = average of SF_t for the same month

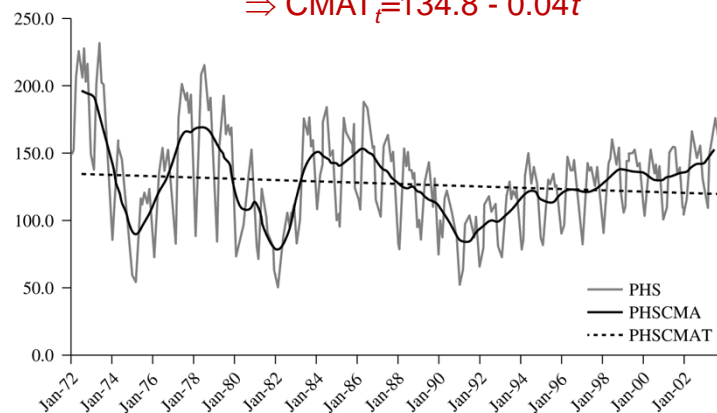
January: 0.71	April: 1.13	July: 1.13	October: 1.10
February: 0.74	May: 1.18	August: 1.11	November: 0.90
March: 1.00	June: 1.18	September: 1.06	December: 0.76

Deseasonalized demand = Actual demand / seasonal index

Estimating the Long Term Trend

Step 3: Regression: $CMA_t = a + b \times t + \varepsilon \Rightarrow a = 134.8, b = -0.04$

$\Rightarrow CMA_t = 134.8 - 0.04t$



Estimating the Cyclical Factor

Step 4: 針對過去資料計算 $CF_t = CMA_t / CMAT_t$

抵銷季節性影響後的銷售，再去除趨勢的影響

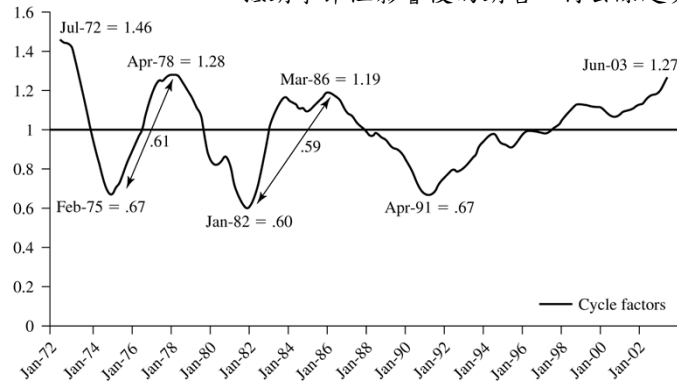


TABLE 6.2 Time-Series Decomposition of Private Housing Starts (c6t2)

1	2	3	4	5	6	7	8	9
Period	Time Index	PHS	PHSMA	PHSCMA	PHSCMAT	CF	SF	SI
Jan-72	1	149.1						0.71
Feb-72	2	152.2						0.74
Mar-72	3	203.9						1.00
Apr-72	4	211.6						1.13
May-72	5	225.8						1.18
Jun-72	6	223.1						1.18
Jul-72	7	206.5	196.38	196.27	134.54	1.46	1.05	1.13
Aug-72	8	228.6	196.17	195.58	134.50	1.45	1.17	1.11
Sep-72	9	203.0	194.98	194.82	134.46	1.45	1.04	1.06
Oct-72	10	216.5	194.66	194.38	134.42	1.45	1.11	1.10
Nov-72	11	185.7	194.11	194.45	134.38	1.45	0.96	0.90
Dec-72	12	150.5	194.79	193.94	134.35	1.44	0.78	0.76
Jan-73	13	146.6	193.08	192.92	134.31	1.44	0.76	0.71
Feb-73	14	138.0	192.76	191.45	134.27	1.43	0.72	0.74
Mar-73	15	200.0	190.14	187.87	134.23	1.40	1.06	1.00
Apr-73	16	205.0	185.59	182.70	134.19	1.36	1.12	1.13
May-73	17	234.0	179.81	177.63	134.16	1.32	1.32	1.18
Jun-73	18	202.6	175.44	172.94	134.12	1.29	1.17	1.18
Jul-73	19	202.6	170.43	167.85	134.08	1.25	1.21	1.13
Aug-73	20	197.2	165.26	164.07	134.04	1.22	1.20	1.11
Sep-73	21	148.4	162.88	159.74	134.00	1.19	0.93	1.06
Oct-73	22	147.1	156.61	154.71	133.97	1.15	0.95	1.10
Nov-73	23	133.3	152.82	149.28	133.93	1.11	0.89	0.90
Dec-73	24	90.4	145.73	143.44	133.89	1.07	0.63	0.76
.
.
.

Overview of Business Cycles

Business cycles描述經濟活動強弱起伏，景氣好的時候，銷售會超過市場趨勢與季節性變化的預期幅度

FIGURE 6.4
The General Business Cycle

A business cycle goes through successive periods of expansion, contraction, expansion, contraction, and so on.

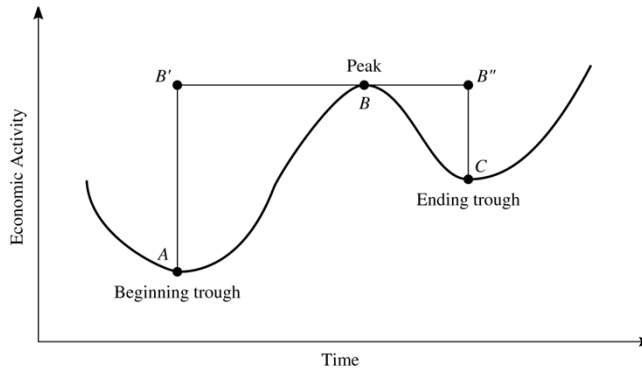


TABLE 6.4
U.S. Business Cycle Indicators

Source: The Conference Board (<http://www.conference-board.org/economics/bci/components.cfm>). Data in this table are from The Conference Board, which produces the U.S. Business Cycle Indicators.

Components of the Composite Indices*

Leading Index

- Average weekly hours, manufacturing
- Average weekly initial claims for unemployment insurance
- Manufacturers' new orders, consumer goods and materials
- Vendor performance, slower deliveries diffusion index
- Manufacturers' new orders, nondefense capital goods
- Building permits, new private housing units
- Stock prices, 500 common stocks
- Money supply, M2
- Interest rate spread, 10-year Treasury bonds less federal funds
- Index of consumer expectations

Coincident Index

- Employees on nonagricultural payrolls
- Personal income less transfer payments
- Industrial production index
- Manufacturing and trade sales

Lagging Index

- Average duration of unemployment
- Inventories-to-sales ratio, manufacturing and trade
- Labor cost per unit of output, manufacturing
- Average prime rate
- Commercial and industrial loans
- Consumer installment credit-to-personal income ratio
- Consumer price index for services

Predicting the Cyclical Factor

Step 1: 運用領先指標或時間數列方法預測未來的CF_t

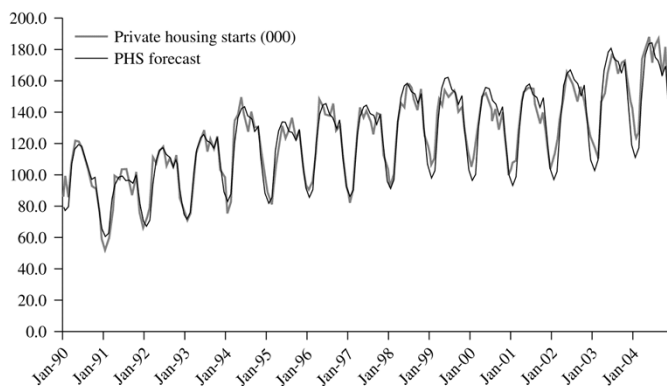


Making Decomposition Forecast

Step 2: $Y = T \times S \times C \times I \Rightarrow Y_t = CMAT_t \times SI_t \times CF_t \quad (I=1)$

FIGURE 6.8
Private Housing Starts (PHS) and a Time-Series Decomposition Forecast (PHSFTSD) for 1990 through 2004 (c6f8)

The actual values for private housing starts are shown by the lighter line, and the time-series decomposition forecast values are shown by the darker line.



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TABLE 6.5
PHS Time-Series
Decomposition
Forecast
(c6t5)

Date	PHS (000)	PHSCMAT	CF	SI	PHS Forecast
Jan-72	149.1			0.708	
Feb-72	152.2			0.743	
Mar-72	203.9			0.999	
Apr-72	211.6			1.128	
May-72	225.8			1.177	
Jun-72	223.1			1.181	
Jul-72	206.5	134.537	1.459	1.128	221.392
Aug-72	228.6	134.499	1.454	1.113	217.660
Sep-72	203.0	134.461	1.449	1.057	206.013
Jul-03	175.8	120.386	1.276	1.128	173.263
Aug-03	163.8	120.348	1.283	1.113	171.834
Sep-03	171.3	120.310	1.291	1.057	164.290
Oct-03	173.5	120.272	1.299	1.104	172.411
Nov-03	153.7	120.234	1.298	0.903	141.021
Dec-03	144.2	120.196	1.300	0.759	118.617
Jan-04	124.5	120.158	1.303	0.708	110.883
Feb-04	126.4	120.120	1.305	0.743	116.468
Mar-04	173.8	120.082	1.302	0.999	156.106
Apr-04	179.5	120.044	1.300	1.128	175.940
May-04	187.6	120.006	1.299	1.177	183.415
Jun-04	172.3	119.968	1.298	1.181	183.778
Jul-04	182.0	119.930	1.292	1.128	174.846
Aug-04	185.9	119.891	1.289	1.113	171.933
Sep-04	164.0	119.853	1.285	1.057	162.915
Oct-04	181.3	119.815	1.282	1.104	169.576
Nov-04	138.1	119.777	1.276	0.903	138.067
Dec-04	141.4	119.739	1.271	0.759	115.517

Forecast

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Department of Logistics Management

Summary

- 繪圖觀察銷售量是否同時受到多種因素的影響。
- Time series decomposition將數據分解以估算trend, seasonality, cycles的影響幅度。
 1. 估算seasonal indexes
 2. 針對去季節化的銷售數據，估算trend
 3. 將去季節化的數據與trend估計值相除，估算Cycle Factors
- 需運用景氣領先指標或其他方法預測cycles的未來變化。
- 預測未來銷售時，再將三者因素重組以進行預測。

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