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Integrated single vendor single buyer model with stochastic demand and variable lead time

M. Ben-Daya[a,*], M. Hariga[b]

[a] Systems Engineering Department, King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia
[b] Industrial Engineering Department, College of Engineering, King Saud University, P.O.Box 800, Riyadh 11421, Saudi Arabia

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Abstract

In this paper, we consider the single vendor single buyer integrated production inventory problem. We relax the assumption of deterministic demand and assume that the lead time is varying lineraly with the lot size. The lead time is composed of a lot size-dependent run time and constant delay times such as moving, waiting and setup times.

A solution procedure is suggested for solving the proposed model and numerical examples are used to illustrate the benefit of integration. A sensitivity analysis is also performed to explore the effect of key parameters on lot size, reorder point, and expected total cost.

Keywords: Integrated model; Single vendor single buyer; Variable lead time; Stochastic demand

1. Introduction

The single vendor single buyer integrated production inventory problem received a lot of attention in recent years. This renewed interest is motivated by the growing focus on supply chain management. Firms are realizing that a more efficient management of inventories across the entire supply chain through better coordination and more cooperation is in the joint benefit of all parties involved. Such collaboration is facilitated by the advances in information technology providing faster and cheaper communication means.

Previous research on the joint vendor buyer problem focused on the production shipment schedule in terms of the number and size of batches transferred between both parties. Most of these models also assumes that demand is deterministic.

One of the first works dealing with the integrated vendor–buyer problem is due to Banerjee (1986). He assumed that the vendor is manufacturing at a finite rate and considered a lot for lot model where the vendor produces each buyer shipment as a separate batch. Goyal (1988) argued that producing a batch which is made up of equal shipments generally produced lower cost but the whole batch must be completed before the first shipment is made. For a review of the literature dealing with the integrated vendor buyer problem prior to 1989, the reader is referred to the paper of

*Corresponding author. Tel.: +9-66-3-860-2968; fax: +9-66-3-860-4426.
E-mail address: bendaya@ccse.kfupm.edu.sa (M. Ben-Daya).

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Goyal and Gupta (1989). Lu (1995) gave an optimal solution to the single vendor–buyer problem assuming equal shipments. Goyal (1995) showed that different shipment size policy could give a better solution. The proposed policy involves successive shipments within a production batch increasing by a constant factor equal to the ratio of the production rate over the demand rate. Recently, Hill (1999) provided another unequal shipment policy for the single vendor single buyer integrated production inventory problem. The policy calls for successive shipments to the buyer, within a single production batch, increasing by a fixed factor. Later, Hill provided an optimal policy for the problem that uses shipments increasing by a fixed factor in the beginning and then remaining constant after a well specified number of shipments.

When the assumption of deterministic demand is relaxed and demand is assumed to be stochastic, lead time becomes an important issue and its control leads to many benefits. The just-in-time (JIT) manufacturing philosophy calls for low lead times to justify the production of small lot sizes. The implementation of such policies in many companies revealed many benefits such as lower investment in inventory, better product quality, less scrap, and reduced storage space requirements (Schonberger, 1982). On the other hand, most inventory models, e.g. the stochastic continuous review model assume that lead time is a given parameter. In a recent paper, Kim and Benton (1995) questioned this assumption and considered the effect of lot size on lead time and safety stock. They established a linear relationship between lead time and lot size based on observations of Karmarkar (1987). They incorporated this lead time lot size relation into the classical stochastic continuous review (Q, s) model. Hariga (1999) modified Kim and Benton's model by rectifying the annual backorder cost and proposing another relation for the revised lot size. Hariga's model is more consistent with JIT's objectives, in the sense that it results in smaller lot sizes. Many researchers looked at the problem of lead time optimization following the papers by Liao and Shyu (1991) and Ben-Daya and Raouf (1994) (see for example Wu, 2001).

In this paper, we consider the single vendor single buyer integrated production inventory problem. We relax the assumption that demand is deterministic and assume that it is stochastic and tackle the lead time issue. We assume a linear relationship between lead time and lot size but take into consideration also nonproductive time in the lead time expression. A solution procedure is suggested for solving the proposed model and numerical examples are used to illustrate the model and explore the effect of key parameters on lot size, reorder point, and expected total cost.

This paper is organized as follows. In the next section, we develop the single vendor single buyer integrated production inventory problem that incorporates stochastic demand and variable lead time. Numerical examples and model results are presented in Section 3. Finally, Section 4 concludes the paper.

2. Model

In this paper, we assume that the buyer is using a continuous review inventory policy. In both deterministic and stochastic continuous review inventory policies, the order quantity and reorder point are often determined under the assumption of a constant lead time. However, from a practical point of view, lead time should be considered as a function of the production lot size. In this section, the classical (Q, s) continuous review inventory policy with deterministic variable lead time is considered for the buyer. In particular, we assume that the lead time is proportional to the lot size produced by the vendor in addition to a fixed delay due to transportation, nonproductive time, etc., that is $L(Q) = pQ + b$.

The relationship between vendor and buyer can be described as follow: the buyer orders a lot of size nQ from the vendor and incurs an ordering cost A. The vendor manufactures the product in lots of size nQ with a finite rate $1/p$ $(1/p > D)$ and incurs a setup cost K. The buyer receives n lots of size Q. He incurs a transportation cost F with each shipment of size Q. The buyer places his order when his on hand inventory reaches a reorder point s after receiving the nth shipment. The

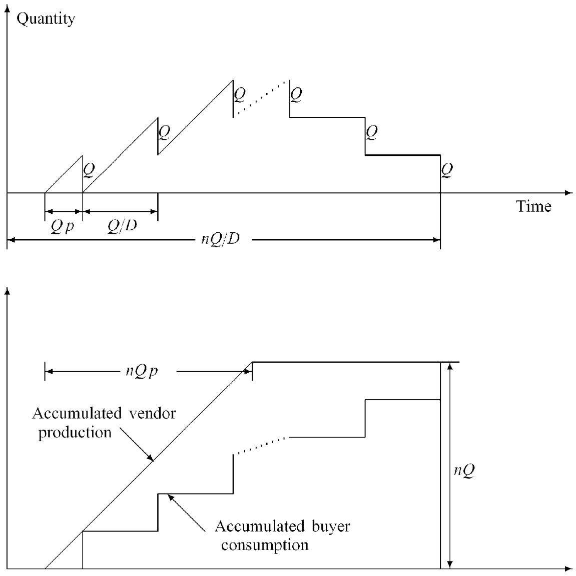

Fig. 1. Inventory of the vendor.

inventory profile for the vendor is depicted in Fig. 1.

The following notation will be used to develop the model:

D	demand rate in units per unit time
$1/p$	production rate in units per unit time
n	number of shipments from the vendor to the buyer
Q	size of equal shipments from the vendor to the buyer
s	reorder point
K	setup cost for the vendor
A	ordering cost incurred by the buyer for each order of size nQ
F	transportation cost for the buyer incurred with each shipment of size Q
h_v	holding cost per unit per unit time for the vendor
h_b	holding cost per unit per unit time for the buyer
S	safety stock
$L(Q)$	lead time $=pQ+b$; where b denotes a fixed delay due to transportation, production time of other products scheduled during the lead time on the same facility, etc.

The total expected cost per unit time for the buyer is given by

$$TC_b = \left(\frac{A}{n}+F\right)\frac{D}{Q} + h_b\left(\frac{Q}{2}+S\right) + \frac{\pi D}{Q}b(s,L(Q)),$$

where

$$b(s,L(Q)) = \int_s^\infty (x-s)f(x)\,dx,$$

where x is the demand during lead time with probability density function $f(x)$.

This model uses the common Hadley–Whitin's (1963) expression ($1/2Q+$ safety stock) to approximate the average inventory level. This approximation depends on the assumption that there is no overshooting (crossing the reorder point) at the time of receiving each shipment (see for example Montgomery and Johnson, 1974, p. 60).

As to the vendor, his total cost per unit time can be obtained from Fig. 1 by subtracting the accumulated buyer consumption from the accumulated vendor production.

$$TC_v = \frac{KD}{nQ} + h_v\frac{Q}{2}[n(1-Dp)-1+2Dp].$$

Consequently, the integrated vendor buyer expected total cost per unit time is given by

$$\begin{aligned} ETC(Q,s,n) = {} & \frac{D}{Q}\left(F+\frac{A+K}{n}+\pi b(s,L(Q))\right) \\ & + \frac{Q}{2}[h_b + h_v[n(1-Dp)-1+2Dp]] \\ & + h_b S. \end{aligned} \tag{1}$$

The problem is to find the number of shipments n, the shipment size Q, and the reorder point s, that minimize the expected total cost (1).

In what follows, we assume that demand during lead time is normally distributed with mean $DL(Q)$ and standard deviation $\sigma\sqrt{L(Q)}$. In this case,

$$S = k\sigma\sqrt{pQ+b},$$

$$\begin{aligned} b(s,L(Q)) &= \int_s^\infty (x-s)f(x,DL(Q),\sigma\sqrt{L(Q)})\,dx \\ &= \sigma\sqrt{pQ+b}\psi(k), \end{aligned} \tag{2}$$

where

$$k = (s-DL(Q))/\sigma\sqrt{L(Q)} \tag{3}$$

and

$$\psi(k) = \int_k^{\infty} (z-k)\phi(z)\,\mathrm{d}z, \tag{4}$$

where $\phi(z)$ is the standard normal probability density function. To simplify notation let

$$H(n) = h_b + h_v[n(1-Dp) - 1 + 2Dp],$$

$$G(n) = F + \frac{A+K}{n}.$$

Consequently, the expression of the expected total cost can be rewritten as

$$ETC(Q,k,n) = \frac{G(n)D}{Q} + \frac{Q}{2}H(n) + h_b k\sigma\sqrt{pQ+b} + \frac{\pi D\sigma\sqrt{pQ+b}}{Q}\psi(k). \tag{5}$$

For fixed n, let us take the derivatives with respect to Q and s and set them to zero.

$$\frac{\partial ETC}{\partial Q} = -\frac{G(n)D}{Q^2} + \frac{H(n)}{2} + \frac{h_b k\sigma p}{2\sqrt{pQ+b}} + \pi D\sigma\psi(k)\frac{\frac{pQ}{2\sqrt{pQ+b}} - \sqrt{pQ+b}}{Q^2} = 0, \tag{6}$$

$$\frac{\partial ETC}{\partial k} = \sigma h_b\sqrt{pQ+b} - \frac{\pi D}{Q}\sigma\overline{F}(k)\sqrt{pQ+b} = 0, \tag{7}$$

where $\overline{F}(k)$ is the complement of the cumulative distribution function, i.e., $\overline{F}(k) = 1 - F(k)$.

After rearranging and simplifying, Eqs. (6) and (7) become

$$\frac{2D}{Q^2}\left[G(n) + \pi\sigma\psi(k)\sqrt{pQ+b}\right] = H(n) + \frac{h_b\sigma p}{\sqrt{pQ+b}}\left[k + \frac{\psi(k)}{\overline{F}(k)}\right], \tag{8}$$

$$\overline{F}(k) = \frac{h_b Q}{\pi D}. \tag{9}$$

It can be easily shown (by taking the second derivative of (7) that the expected total cost function (1) is convex in k. However, the cost function may not be convex in Q. Eq. (6) can be rewritten as

$$Q = \sqrt{2D\frac{G(n) + \pi\sigma\psi(k)\sqrt{pQ+b}}{H(n) + \frac{h_b\sigma p}{\sqrt{pQ+b}}\left[k + \frac{\psi(k)}{\overline{F}(k)}\right]}} \tag{10}$$

and the value of k can be obtained from

$$\overline{F}(k) = \frac{h_b Q}{\pi D}. \tag{11}$$

The following iterative procedure can be used to find an approximate solution to the above problem:

Algorithm.

Step 0: Set $ETC^* = \infty$ and $n = 1$

Step 1: Compute $Q = [\sqrt{2DG(n)/H(n)}]$, where $[x]$ is the nearest integer to x.

Step 2:

- Find k from (9)
- Compute $\psi(k)$ using (4)

Step 3:

- Compute Q' using (10)
- Set $Q' = [Q']$

Step 4:

- If $|Q' - Q| = 0$, compute $ETC(Q,n)$ and go to **Step 5**
- If $|Q' - Q| > 0$, set $Q \leftarrow Q'$ and go to **Step 2**

Step 5:

- If $ETC^* \geqslant ETC(Q,n)$ then $ETC^* \leftarrow ETC(Q,n)$, $Q^* \leftarrow Q, s^* \leftarrow s$, Set $n \leftarrow n+1$ and go to **Step 1**
- Otherwise, $n^* \leftarrow n-1$ and **stop**.

3. Numerical example

In this section, we illustrate the model with the example given in Table 1.

The results for the example given in Table 1 are summarized in Table 2. As expected, lower transportation costs justify lower batches, lower reorder points and lower total expected cost.

Table 1
Example data

D	=	1000 units	demand rate on the buyer
σ	=	5 units	standard deviation of demand
$P = 1/p$	=	3200 units	production rate for the vendor
K	=	\$ 400	setup cost incurred by the vendor
F	=	\$ 25	transportation cost incurred by the buyer
A	=	\$ 50	ordering cost incurred by the buyer
h_b	=	\$ 5	holding cost per unit per unit time for the buyer
h_v	=	\$ 4	holding cost per unit per unit time for the vendor
π	=	\$ 100	backorder cost for the buyer
b	=	0.01	fixed delay due to transportation

Table 2
Effect of key model parameters

		Q	n	s	ETC
F	35	142	4	57	2084.82
	25	115	5	49	2007.77
	15	95	6	42	1912.53
b	0.100	115	5	141	2018.65
	0.010	115	5	49	2007.77
	0.001	115	5	39	2006.94
h_b	5	115	5	49	2007.77
	7	95	6	42	2117.42
	10	73	8	35	2251.36

The results obtained for various values of nonproductive lead time shows that higher lead times lead to higher reorder points and higher total cost. However, the effect on the batch size is minimal.

Table 2 shows also that total cost, reorder points and batch sizes are sensitive to changes in inventory holding cost.

It is also informative to compare the integrated system solution with the independent solutions. The following scenarios are considered:

1. The buyer observes the demand and uses an optimal (Q, r) policy. We assume, for the purpose of comparison, that the lead time is that of the optimal integrated solution. Let Q_b be the optimal order quantity of the buyer. The vendor uses the EPQ model based on the buyer average demand but produces lots of size Q_b.

Table 3
Comparison of integrated and independent solutions.

Scenario	Vendor		Buyer			System Cost
	Q_v	Cost	Q_b	r	Cost	
1	173	2658.14	173	49	881.73	3539.87
2	447	1788.85	477	48	1298.08	3086.93
3	447	1788.85	141	49	722.90	2511.75
Integrated	575	–	115	49		2007.77

2. The vendor produces lots of size Q_v following the optimal EPQ solution and based on the average external demand. The vendor use a (Q, r) policy but orders lots of size Q_v.
3. Both parties observe the same external demand. However, they act independently. The vendor uses the optimal EPQ policy and the buyer uses the optimal (Q, r) inventory policy.

The results for these three scenarios are summarized in Table 3.

Note the significant cost reduction due to the integration of vendor and buyer decisions. The integrated solution is superior even to the scenario where each party chooses its independent optimal policy, scenario 3.

4. Conclusion

In this paper, we considered the single vendor single buyer integrated production inventory problem. Previous work on this problem focused on the production shipment schedule in terms of the number and size of batches transferred between both parties. Previous models also assumed that demand is deterministic. Here, we assume that demand is probabilistic and the lead time is variable and depends on lot size and other delays, such as transportation time. A simple procedure is suggested to obtain an approximate solution of the proposed model. Examples are used to illustrate the model and explore the effect of important parameters on the production schedule and total expected cost.

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