Proceedings of DETC'00
ASME 2000 Design Engineering Technical Conferences
and Computers and Information in Engineering Conference
Baltimore, Maryland, September 10-13, 2000

DETC2000/DAC-14531

MODIFIED SEQUENTIAL PROGRAMMING FOR FEASIBILITY ROBUSTNESS OF CONSTRAINED DESIGN OPTIMIZATION

Jyh-Cheng YU*, Wen-Chung Ho

Department of Mechanical Engineering
National Taiwan University of Science and Technology
Taipei 106, Taiwan, R.O.C.

.

ABSTRACT

Design parameters are subject to variations of manufactures, environments, and applications, which result in output deviations and constraint uncertainties. Quality products must perform to specifications despite these variations. Conventional constrained optimum may not be statistically feasible due to design variations. This paper addresses design variation characteristics and proposes a design procedure to ensure feasibility robustness in design optimization. Product life cycles often affect design variables with characteristic patterns. The Design Variation Hyper Sphere (*DVHS*) is presented using the concept of statistical joint confidence regions and decoupling techniques to characterize the coupled variations of design variables. The pattern represents the possible design dispersions at a specified probability, which is a hyper sphere for normal variables. The radius of the hyper sphere is determined by the feasibility requirement. The proposed robust optimization algorithm, *SROP*, introduces *DVHS* to the sequential quadratic programming, and modifies the feasible region to accommodate the activity uncertainty. The procedure ensures the design feasibility without over sacrificing the performance optimality. The design of a helical spring serves as an illustrative example of the proposed procedure.

Keywords: Variation Hyper Sphere, Taguchi, Constraint Uncertainty, Statistical Optimization, Robust Design, Sequential Quadratic Programming,

1. INTRODUCTION

Variations of manufacture, environment, and application cause design parameters to deviate from nominal values, which introduce performance fluctuations. Conventional engineering opts to use tolerance control to reduce the output deviation, which, however, often leads to a higher manufacturing cost. Quality design should deliver the target performance despite these variations.

Parameter design, advocated by Taguchi [1], minimizes the performance sensitivity to variations rather than controlling the sources. Taguchi's method features signal-to-noise ratio, orthogonal array experiments, and analysis of variance to perform a two-step design optimization. Many studies apply Taguchi's concept to nonlinear programming problems to minimize the variations of nominal optimum. d'Entremont & Ragsdell [2] adopted the concept of quality-loss to modify the objective function. Sandgren [3] and Sundaresan et al. [4] formulated the design objective as functions of weighed mean and deviation of performance. However, Taguchi's approach to experimental design does not clearly address potential interactions between controllable factors, which might result in estimation errors of performance tolerance. D'Errico & Zaino [5] presented a modified approximation using the Gaussian-Hermite quadrature integration. Yu & Ishii [6] proposed the Fractional Quadrature Factorial to estimate the performance mean and the robustness for applications with significant interaction and nonlinear effects.

The constraint uncertainty due to parameter variations is another issue in robust optimization. The conventional, actively constrained, optimum may not be statistically feasible. Parkinson et al. [7] explored the influence of correlated constraints on feasibility and advocated a two-step solution to modify the feasible region. Sundaresan et al. [8] compared the efficiency of three different methods that incorporate variations in constraints. Most of these studies use worst-case

* Corresponding author

analysis and fall short of addressing the variation characteristics. Yu & Ishii [9] proposed the concept of Manufacturing Variation Patterns (*MVP*) to characterize the coupled variations of design variables. The recognition of variation patterns is essential to the estimate of performance distribution and design feasibility in design optimization.

This paper investigates the influence of correlated variations on the identification of a constrained optimum. The concept of *MVP* is extended to include the variations of controllable variables and uncontrollable parameters throughout the product life cycle. Manufacturing errors, operation temperature, and mechanical wear often affect design variables with characteristic patterns. The assumption of independence among variables is no longer valid. The "variation pattern" will affect the performance and feasibility robustness of a constrained optimum. This paper aims to develop an algorithm to identify the constrained robust optimum based on the pattern.

2. ROBUST OPTIMIZATION

Engineering optimization seeks the design with the best objective in the feasible region. Design objective and constraints are functions of design variables that are subject to product life cycle variations. The nominal optimum may not contain the best mean objective and the least sensitivity to design variations. Also, product life cycle variations affect constraint activity. The conventional optimum with active constraints is not statistically feasible. There will be a large portion of unsatisfactory occurrence when the design is in production. Therefore, the robust optimum should consider the two following issues:

(1) Feasibility Robustness

A constrained optimum should assure feasibility despite parameter variations. One resolution is to move the design toward the feasible region. The moving distance is determined by the acceptable probability of infeasible occurrence. The joint distribution region of design variables could represent the design variation pattern, which often appear as an ellipsoid [9]. The shape of the variation pattern depends on the correlation levels of design variables and the distribution probability inside the pattern. The locus of the centroid of the variation pattern tangent to the nominal inequality constraint $g_i(X)$ composes the Robust Inequality Constraint $g_i^R(X)$. Confining the designs inside the modified feasible region bound with $g_i^R(X)$ in the search process will provide designs with feasibility robustness (Figure 1).

(2) Performance Robustness

A robust optimum requires the best mean objective and the least sensitivity to design variations. A conventional search for an optimum considers only nominal output. However, the use of output mean and deviation can better describe output characteristics. Figure 2 shows that design x_1 has a smaller nominal output but much larger output deviation than design x_2. In other words, design x_2 has less sensitivity to variation than design x_1. Besides, the mean output and the nominal output may differ in case of asymmetric distribution as shown in y_{3n} of Figure 2. Optimization of nominal output will not ensure performance quality. The modified objective function consisting of output mean and deviation is an intuitive approach for performance robustness.

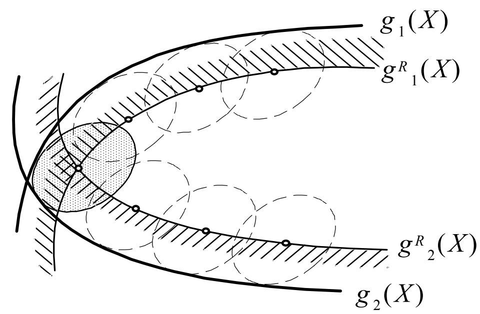


Figure 1. Modification of constraint using design variation pattern

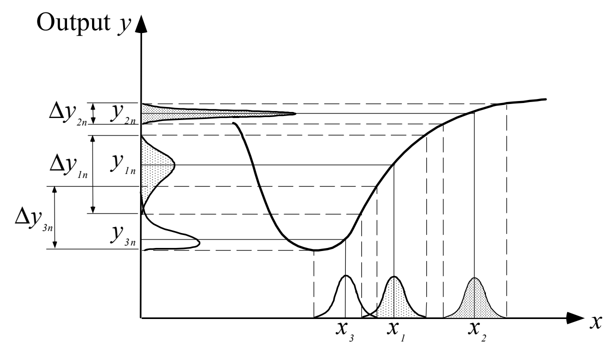
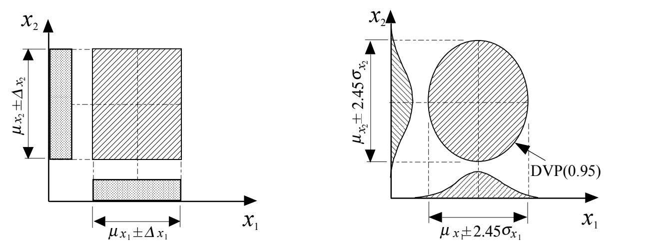


Figure 2. Output distributions of various designs

3. DESIGN VARIATION PATTERN

Manufacturing errors and application diversities often induce the dispersion of design variables with characteristic patterns. Here, the design variables include controllable variables whose values can be selected by designers and uncontrollable parameters whose values are fixed as part of the specifications. Conventional worst case regions (WCR) assume independence of design variables, which fails to explain correlated dispersions. The dispersion pattern should be the possible combination of the variables at the specified probability. The *Design Variation Pattern* (*DVP*) extends the concept of *Manufacturing Variation Pattern* [9] and uses the statistical joint confidence region to characterize the coupled variations of design variables.

The shapes of *DVP* are affected by the various factors in a product life cycle such as manufacturing processes and application temperatures. These "variation patterns" are particularly important in net shape manufacturing and heat treated parts where the dimensional errors are largely due to material shrinkage that simultaneously affects multiple variables. Most literature uses the worst case region (WCR) to describe the distribution of design variables [7,8]. The shape of WCR is a rectangular-hyper-solid, which neglects the possible correlation among variables. The corners of WCR, which represent the distribution with extreme low possibility of occurrence, might lead to over conservative design in constrained problems. This paper applies statistical multivariate regression to derive the confidence region of design variation. Figure 3(b) is an example of *DVP* of two independent variables.



(a) Worst Case Region (b) *DVP*(1-α) confidence region

Figure 3. Typical distribution patterns for *WCR* and *DVP*

Different product life cycles introduce variations to design variables *X*. (Eq. 1) and (Eq. 2) represent the variance-covariance matrix and the correlation efficient. The correlation coefficient ρ_{ij} measures the strength of the linear association between two variables, x_i and x_j.

$$\Sigma = Exp\left[(X - X_c)(X - X_c)^T\right] = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1p} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{p1} & \sigma_{p2} & \cdots & \sigma_{pp} \end{bmatrix} \quad \text{(Eq. 1)}$$

where $X=(x_1, \ldots x_p)^T$

$$X_c = \left[Exp(x_1), Exp(x_2), \cdots, Exp(x_p)\right]^T = \left[\mu_1, \mu_2, \cdots, \mu_p\right]^T$$

$$\rho_{ij} = \frac{\sigma_{ij}}{\sqrt{\sigma_{ii}}\sqrt{\sigma_{jj}}} \quad \text{(Eq. 2)}$$

If x_i are normal, and their means μ_i and the covariance matrix Σ are given, $(X- X_c)^T \Sigma^{-1}(X- X_c)$ is Chi-square distributed [10]. The boundary of the (1-α)×100% confidence region of *X* is defined as Design Variation Pattern, denoted *DVP*(1-α).

$$(X - X_c)^T \Sigma^{-1} (X - X_c) = \chi^2_{p,\alpha} \quad \text{(Eq. 3)}$$

where $\chi^2_{p,\alpha}$ is the chi-square value of *p* degrees of freedom, which leaves $\alpha \times 100\%$ in the upper tail of distribution.

The left side of (Eq. 3) can be reformulated as follows using matrix operations:

$$(Y\text{-}Y_c)^T \Lambda^{-1}(Y\text{-}Y_c) = \chi^2_{p,\alpha} \quad \text{(Eq. 4)}$$

where $Y=E^TX$

$Y_c=E^TX_c$

$$E = \left[\vec{e}_1 \quad \vec{e}_2 \quad \cdots \quad \vec{e}_p\right]$$

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & \cdots & 0 & \lambda_p \end{bmatrix}$$

$\vec{e}_i$ is the unit eigenvectors of matrix Σ

λ_i is the eigenvalues of matrix Σ

The function of *DVP* can then be represented as follows:

$$\frac{(y_1 - y_{1c})^2}{\lambda_1 \chi^2_{p,\alpha}} + \frac{(y_2 - y_{2c})^2}{\lambda_2 \chi^2_{p,\alpha}} + \cdots + \frac{(y_p - y_{pc})^2}{\lambda_p \chi^2_{p,\alpha}} = 1 \quad \text{(Eq. 5)}$$

Equation (Eq. 5) reveals that *DVP* is an ellipsoid centered at X_c. The axes of the ellipsoid lie in the direction of the eigenvectors, $\vec{e}_i$, of Σ. The length of the principal axes are equal to $\sqrt{\chi^2_{p,\alpha}\lambda_i}$, where λ_i are the eigenvalues of Σ and α is the confidence level. Figure 4 shows a *DVP* of two correlated variables. The size of *DVP* increases with the requirement of confidence (1-α)×100%. The correlation among design variables will make *DVP* an oblique ellipsoid. The correlation level affects the tile direction.

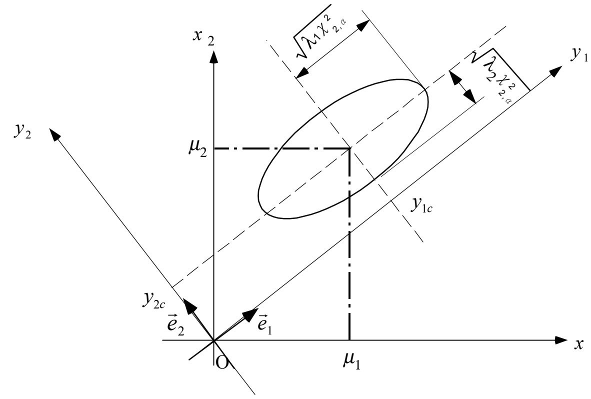


Figure 4. Typical shape of two-dimensional *DVP*

4. DESIGN VARIATION HYPER-SPHERE

To ensure feasibility robustness, inequality constraints are modified toward feasible regions according to *DVP* (Figure 1). The shape of *DVP* varies with the correlation level among variables. The distance between the nominal design and the boundary of the variation pattern is not constant, which increases the computational difficulty to determine constraint modification. Our scheme adopts the coordination transformation to decouple the correlation and to transform the varying "statistical variation distance" to a uniform "Design Variation Radius". The result will greatly simplify the process of constraint modification. Since covariance matrix Σ is real symmetric, there exists an orthogonal matrix E to diagonalize Σ.

$$E^{-1}\Sigma E = E^T \Sigma E = \Lambda$$

$$\Sigma = E\Lambda E^T \qquad \text{(Eq. 6)}$$

where the definitions of E and Λ are the same as in (Eq. 4)

Reformulate the function of *DVP* (Eq. 3) using (Eq. 6).

$$\left[E^T(X - X_c)\right]^T \Lambda^{-1}\left[E^T(X - X_c)\right] = \chi^2_{p,\alpha} \qquad \text{(Eq. 7)}$$

Σ is positive definite due to the properties of real symmetric matrices. Therefore, the diagonal elements of matrix Λ are all positive real. There exists a diagonal matrix V, such that

$$\Lambda = VV \qquad \text{(Eq. 8)}$$

Let $Y=E^TX$ and substitute (Eq. 8) into (Eq. 7).

$$(Y - Y_c)^T V^{-1}V^{-1}(Y - Y_c) = \chi^2_{p,\alpha} \qquad \text{(Eq. 9)}$$

Define the standardized variables Z

$$Z = V^{-1}Y = V^{-1}E^T X \qquad \text{(Eq. 10)}$$

$$X=EVZ \qquad \text{(Eq. 11)}$$

The *DVP* is reformulated as a function of variable Z, which is.

$$(Z - Z_c)^T (Z - Z_c) = \chi^2_{p,\alpha} \qquad \text{(Eq. 12)}$$

Z_c is the standardized value of nominal design X_c that is also the means of X. The variation pattern in the standardized Z domain becomes a hyper-sphere that represents the variation space of design Z_c. (Eq. 12) is defined as the Design Variation Hyper-Sphere (*DVHS*) centered at Z_c. $\sqrt{\chi^2_{p,\alpha}}$ is termed the Design Variation Radius at the confidence of $(1-\alpha)\times 100\%$.

5. DESIGN OPTIMIZATION FOR FEASIBILITY ROBUSTNESS

Life-cycle variations introduce dispersions to design variables and propagate to design constraints. Robust optimization applies statistical techniques to refine the definitions of equality and inequality constraints. A constrained optimum should be statistically feasible regardless of the possible dispersion of design variables (Figure 1). In other words, feasibility robustness requires no constraint violation for the entire design variation pattern. Two-stage optimization is often used to ensure feasibility. After locating the nominal optimum, the second stage of optimization modifies design constraints to accommodate parameter variations, and searches for the robust optimum in the modified feasible region. The amount of modification depends on the shape of the variation pattern that is an ellipsoid for normal variables.

If the variation distance is constant despite the orientation of design, the modification process will be greatly simplified. This paper applies the concept of Design Variation Hyper-Sphere (*DVHS*) to sequential quadratic programming, and proposes the algorithm of Sequential Robust Optimization. The algorithm can readily apply to the two-stage optimization or search for the constrained optimum directly.

5.1. Sequential Quadratic Optimization for Feasibility Robustness

Sequential quadratic programming (*SQP*) linearizes the constraint function at the initial design using the first order Taylor's expansion. The objective function, on the other hand, is approximated using a second order Taylor's expansion. The optimum of the simplified problem can be solved for much easily. The process iterates the new design until the convergence of the result, which will approach the constrained optimum of the nonlinear problem.

The proposed scheme of Sequential Quadratic Optimization for feasibility robustness modifies the linearized constraints in *SQP* to accommodate the design variations. The first step is to derive the *DVHS* from the statistical design data. The constraints and the objective are transformed to the functions of standardized variables Z.

$$f(X)=f[p(Z)]=F(Z) \qquad \text{(Eq. 13)}$$

$$g_i(X)=g_i[p(Z)]=G_i(Z) \qquad \text{(Eq. 14)}$$

where $X=p(Z)=EVZ$

$Z=[z_1, z_2, \ldots, z_p]^T$

The Design Variation Radius is introduced to *SQP* to modify the linearized constraints to ensure feasibility robustness. The modification process is as follows:

The standardized constraints $G_i(Z)$ are linearized at initial design Z_0.

$$G_{iL}(Z) = \nabla G_i(Z_0)^T (Z - Z_0) + G_i(Z_0) \qquad \text{(Eq. 15)}$$

The linearized constraints $G_{iL}(Z)$ offset along the gradient direction, $\nabla G_i(Z_0)$, for variation radius r. As shown in Figure 5, all the points Z on $G_{iL}(Z)$ and the corresponding points Z_R on the shifted line $G^R_{iL}(Z_R)$ have the following relation.

$$Z = Z_R + r\left(\frac{\nabla G_i(Z_0)}{\left|\nabla G_i(Z_0)\right|}\right) \qquad \text{(Eq. 16)}$$

where Z_0 is the initial design

r is the Design Variation Radius of the *DVHS*

Z_R is the set of points on $G_{iL}(Z)$ offset along the negative gradient direction for the distance of r.

Substitute (Eq. 16) into (Eq. 15) and the linearized constraint after modification is:

$$G_{iL}^R(Z_R) = \nabla G_i(Z_0)^T (Z_R - Z_0) + r\frac{\nabla G_i(Z_0)^T \nabla G_i(Z_0)}{|\nabla G_i(Z_0)|} + G_i(Z_0) \quad \text{(Eq. 17)}$$

Since Z_R is only a variable, we can replace Z_R with Z and simplify the equation as follows:

$$G_{iL}^R(Z) = \nabla G_i(Z_0)^T (Z - Z_0) + r|\nabla G_i(Z_0)| + G_i(Z_0) \quad \text{(Eq. 18)}$$

Comparing (Eq. 18) and (Eq. 15), we obtain

$$G_{iL}^R(Z) = G_{iL}(Z) + r \cdot |\nabla G_i(Z_0)| \quad \text{(Eq. 19)}$$

where $r|\nabla G_i(Z_0)|$ is the amount of modification for the linearized constraint to accommodate design variations.

Sequential Robust Optimization (*SROP*) combines the proposed scheme and the *SQP* to modify the linearized constraint using (Eq. 19) in each iteration. The procedure ensures the searched design will satisfy the (1-α)×100% confidence of feasibility. The application of the proposed algorithm is straightforward, and can be readily integrated with conventional nonlinear programming methods.

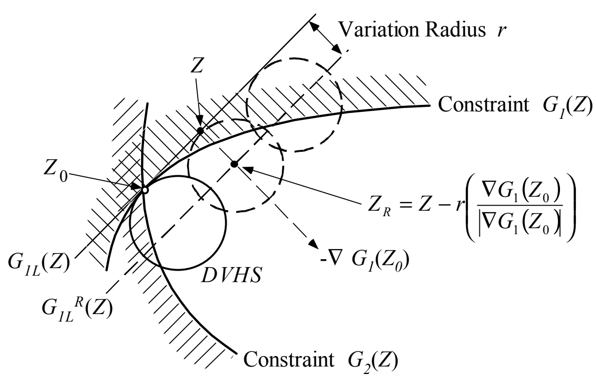


Figure 5. Modification of linearized constraints using *DVHS*

5.2. Numerical Example for Robust Feasibility

Consider a simple two-dimensional optimization as follows:

Minimize $y(x_1, x_2) = -2x_1 - x_2$

Subject to $g_1 = x_1^2 + x_2^2 - 25 \le 0$

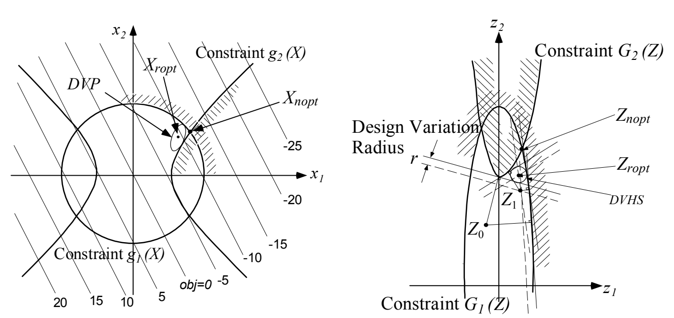
$g_2 = x_1^2 - x_2^2 - 7 \le 0$

Assume the variables are normal with the standard deviations σ_1=0.005 and σ_2=0.3, and the correlation coefficient between x_1 and x_2 is 0.75. The confidence requirement of the feasibility is no less than 95%. Figure 6(a) represents the nominal optimum X_{nopt} and the robust optimum X_{ropt} with *DVP*. The search of X_{ropt} is more complicate because the shape of *DVP* is an oblique ellipse. Figure 6(b) shows the optimization problem in the space of standardized variables. A couple of iterations before convergence are shown in Figure 6(b). First, the standardized constraints are linearized at Design Z_0. Because the variation pattern becomes a circle in the Z space, the modification of the linearized constraints simply shifts the lines toward the feasible region by the distance of the Design Variation Radius r. Design Z_1 is found for the modified constraints and the response contour. A similar procedure is applied to Z_1 to reach the robust optimum Z_{ropt}. Table 1 shows the results of the nominal optimization, Parkinson's method (statistical variation coefficient k=2), and the Sequential Robust Optimization. The probability of feasibility is estimated using Monte Carlo simulations.

Table 1. The Nominal Optimum and the Robust Feasible Optimum

	Nominal Optimization	Parkinson's Method	Sequential Robust Optimization
Variable x_1	4.0	3.5084	3.496
Variable x_2	3.0	3.0005	2.906
Nominal Objective	-11.0	-10.017	-9.898
Probability of Unfeasibility	0.9524	0.0518	0.0307

Both the results of Parkinson's method and *SROP* move the designs toward the inside of the feasible region to ensure robustness. Though the objectives are higher than the nominal optimum, the probabilities of unfeasibility are greatly reduced. The confidence specification of the *DVHS* is 95%, which represents that the statistical probability outside the *DVHS* is 5%. However, the *DVHS* is usually tangent to some of the constraints, and the actual probability outside the feasibility will be less than 5%. We can always trade off between the objectives and the feasibility by adjusting α of *DVHS*.



(a) Original Contour (b) Standardized Contour

Figure 6. The standardization of the variation pattern and the constraints

6. Computer Aided Robust Optimization

The window application *SROP* is presented by integrating the proposed optimization scheme for feasibility robustness and the nonlinear programming software *DOT* [11]. Figure 7 summarizes the flow chart of the Sequential Robust Optimization (*SROP*). *SROP* uses Multimedia Toolbook [12] to compose the interface (Figure 8) and Borland C for the mathematics programming.

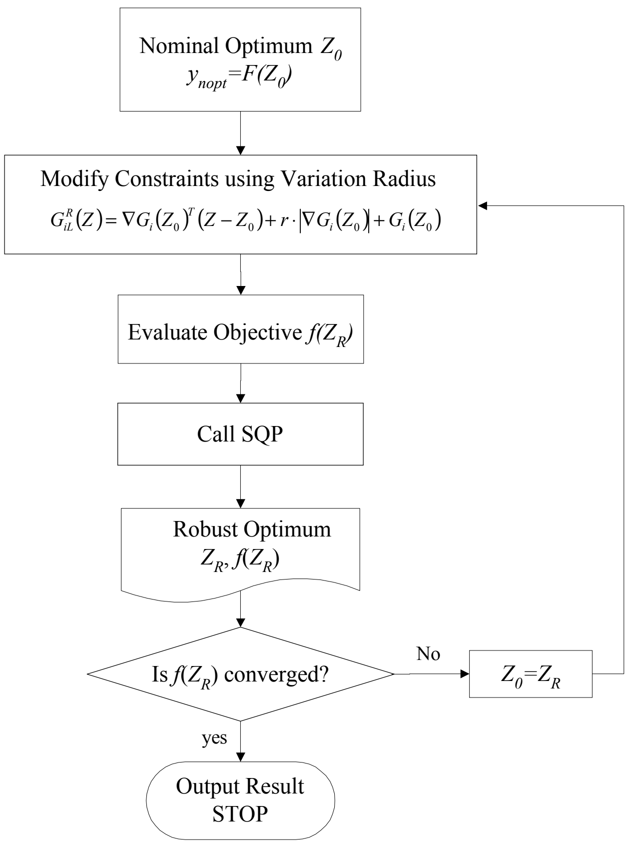


Figure 7. The Sequential Robust Optimization flow chart

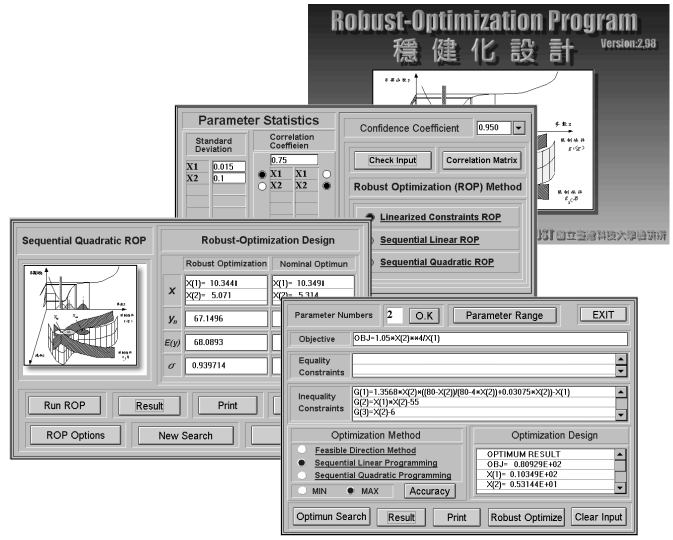


Figure 8. Program interface of *SROP*

7. Design of a Helical Spring

This design example of a helical spring [13] will illustrate the engineering application of *SROP*. The length of the spring is compressed from h_{free} to h_0 due to a static load F_0. A released compression load F_{rc} is also applied to the spring, which introduces a reciprocal deformation of δ_{rc}. Given $h_0 = 58$ (mm), $\delta_{rc} = 5$ (mm), $h_{free} = 66$ (mm), and outside diameter $D_o \leq 27$ (mm), find the spring design with maximum allowable static load F_0.

The characteristics of the selected material are as follows:

Modulus of rigidity $G = 8.4\times10^3$ (MPa)
Allowable stress $s_w = 84$ (MPa)
Fatigue strength $s_e = 42$ (MPa)
Safety factor $SF = 1.1$

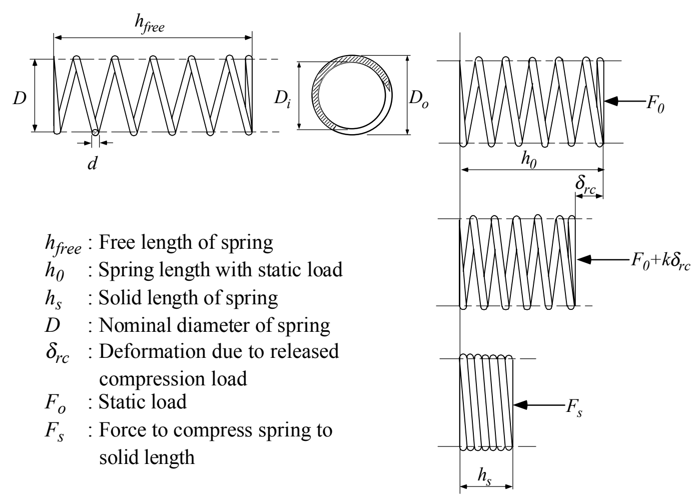


Figure 9. Illustration of the helical spring

The spring example can be formulated as the following optimization problem. The first inequality constraint is a fatigue stress requirement. s_m is the mean stress and s_a is the alternative stress of the spring due to the fluctuating load. The second constraint represents the size limit for the solid length of the spring. The third constraint represents the limit for the spring outside diameter.

$$\text{Maximize}\quad F_0 = K_s\delta = \left(\frac{Gd^4}{8D^3n}\right)\left(h_{free} - h_0\right)$$

$$\text{Subject to}\quad \left(\frac{s_m - s_a}{s_w} + \frac{s_a}{s_e}\right) \le \frac{1}{SF}$$

$$nd \le (h_0 - \delta_{rc})$$

$$D + d \le 27$$

$$D,\ d,\ n > 0$$

The design variables D, n, and h are subject to manufacturing errors and fluctuations of operational temperature. The uncontrollable parameters G, s_w, and s_e are subject to uncertainties of material property. These variations result in the distribution of design objective. For the simplicity of illustration, this paper only considers the variation of geometrical variables. The standard deviations of the dimensional variables are estimated as follows:

$\sigma_n = 0.015$

$\sigma_d = 0.1$ (mm)

$\sigma_D = 0.25$ (mm)

These variations are possibly correlated due to a manufacturing process [9], such as heat treatment of the coil. For instance, the coil diameter and the spring diameter are positively correlated due to temperature effects, while the coil number might fluctuate in the opposite direction due to stress relief. This example assumes the correlation coefficients $\rho_{nd} = -0.75$, $\rho_{nD} = -0.75$, $\rho_{dD} = 0.4$.

Table 2 shows the results using conventional peak optimization and the proposed scheme (α =0.75). The probability of constraint violation is estimated with 100000 samplings using Monte Carlo simulation to evaluate the design feasibility. The expected output and the standard deviation are calculated using the probability integration.

The nominal optimum contains a larger expected output. However, the standard deviation of the nominal optimum is high, and 80 percent of the design distribution falls in the unfeasible region. *SROP* moves the designs toward the interior of the feasible region to accommodate the uncertainty of variables, and reduce the probability of unfeasibility to only 5%. The standard deviation of robust design is reduced by 13% compared with the nominal optimum. The cross sections of the contour plot (Figure 10) show that the change of the objective is gradual. When the nonlinearity in the vicinity of the nominal optimum increases, the improvement using *SROP* will be more significant.

Table 2. Design comparison of helical springs

	Nominal Optimum	*SROP*
Number of coil n	9.466	9.654
Coil diameter d (mm)	5.599	5.30
Nominal spring diameter D (mm)	21.401	21.084
Nominal output (N)	88.985	73.229
Expected response (N)	89.114	73.353
Standard deviation of response (N)	5.911	5.121
Unfeasibility probability (%)	80.2	5.4

8. CONCLUSION

This paper addresses the influence of design variations on feasibility uncertainty in design optimization. We propose a robust optimization scheme, *SROP*, which adopts the concept of Design Variation Pattern (*DVP*) in the modification of design constraints. The concept of *DVP* is transformed to the Design Variation Hyper-Sphere that is introduced to the nonlinear programming algorithm *SQP*. *SROP* provides an optimal design with satisfactory feasibility despite possible correlation among variations. The *SROP* program is presented to facilitate the application of robust optimization. The spring design example demonstrates the effectiveness of the proposed scheme. The robust design of a helical spring improves the output deviation and the design feasibility at the minimum expense of mean output.

ACKNOWLEDGEMENT

This research is funded by the National Science Council of the Republic of China under grant NSC 86-2212-E-011-011.

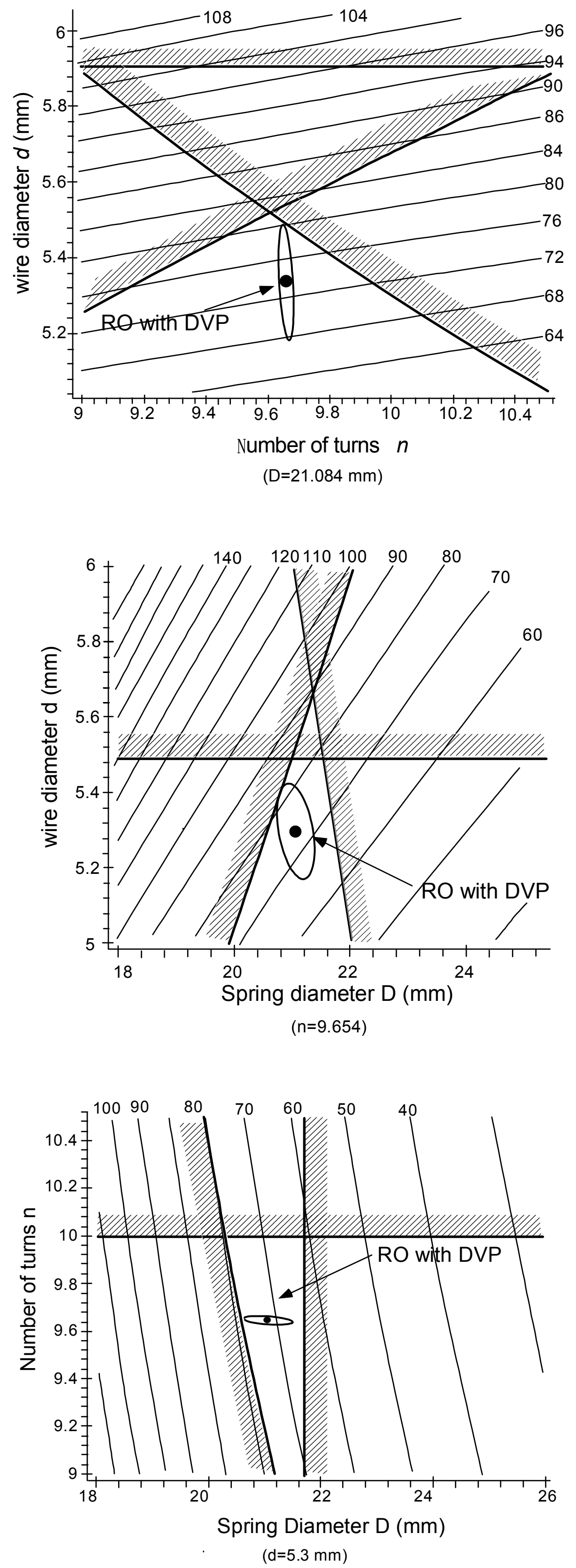


Figure 10. Cross-section contour plots and constraints

REFERENCE

[1] Taguchi, G., 1978, "Performance Analysis Design." *International Journal of Production Research*, 16, pp.521-530.

[2] d'Entremont, K.L. and Ragsdell, K.M, 1988, "Design for Latitude Using TOPT.", *ASME Advance in Design Automation*, DE-Vol. 14, pp.265-272

[3] Sandgren, E., 1989. " Multi-Objective Design Tree Approach for Optimization Under Uncertainty." *Proceedings of the 1989 ASME Design Automation Conference*. Montreal Canada, Vol. 2, pp.249-255.

[4] Sundaresan, S. , Ishii, K. , and Houser, D. R., 1991, "A Procedure Using Manufacturing Variance To Design Gears With Minimum Transmission Error", *ASME Journal of Mechanical Design* , Vol. 13, pp.318-324

[5] D'Errico J. R. and Zaino, Jr. N. A., 1988, "Statistical Tolerancing Using a Modification of Taguchi's Method.", *Technometrics*, 30(4), pp.397-405

[6] Yu, J., and Ishii, K., 1998, "Design Optimization for Robustness Using Quadrature Factorial Models" *Engineering Optimization*. Vol. 30, pp.203-225.

[7] Parkinson, A. , Sorensen, C., and Pouhassan, N., 1993, "A General Approach for Robust Optimal Design.", *ASME Journal of Mechanical Design*, Vol. 115, pp.74-80

[8] Sundaresan, S., Ishii, K. and Houser, D. R., 1993, "A Robust Optimization Procedure With Variations on Design Variables and Constraints", *Proceedings of ASME Advance in Design Automation Conference*. Sept. 20-22, Albuquerque, NM. Vol. 65-1, pp.379-386

[9] Yu, J. and Ishii, K. , 1998, "Design for Robustness Based on Manufacturing Variation Patterns", *ASME Journal of Mechanical Design*, Vol. 120, p.196-202

[10] Johnson, R. A. and Wichern, D. W., 1988, *Applied Multivariate Statistical Analysis*, Englewood Cliffs, New Jersey: Prentice-Hall

[11] Vanderplaats, Miura & Associate, 1993, *Dot User Manual*, Version 4.0, VMA Engineering

[12] *Multimedia ToolBook* Version 3.0a, 1994, Asymetrix Corporation

[13] Fox, R. L., 1971, *Optimization Methods for Engineering Design*, Addison-Wesley, Reading, MA, pp.35-36.