An Application of the Multivariate Student-*t* Distribution-Based EC-DCC Model on Hedging Effectiveness in Stock Index Futures Markets

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Abstract

This paper extends Engle's (2002) multivariate normal distribution based dynamic conditional correlation GARCH (DCC) model to a multivariate Student-*t* distribution based error correction dynamic conditional correlation GARCH (EC-DCC) model in order to investigate dynamic interactions between stock and futures markets in the FTSE 100 and Nikkei 225. We then apply this extended model to estimate the optimal time-varying hedge ratios. A hedging efficiency comparison among our model and other usually used models is conducted to shed light on the time-varying conditional correlation coefficients and the multivariate Student-*t* distribution settings. The empirical results of the hedging efficiency comparisons find that the multivariate Student-*t* distribution based EC-DCC model performs best in the FTSE 100 and Nikkei 225. This is because our empirical model takes all three important characteristics in the interaction between the stock and futures markets into account simultaneously: the long-term trend, the time-varying conditional correlation coefficients as well as volatility, and the fat tail or leptokurtic characteristic.

Keywords: Optimal hedge ratios, Hedging effectiveness, Multivariate Student-*t* distribution, Error correction, DCC model

JEL classification: C32, C53, G15

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I. Introduction

Ever since the Value-line Stock Index Futures contract went public in 1982 in the Kansas City Board of Trade (KCBT), numerous stock index futures contracts have continued to appear and have helped investors satisfy their arbitraging, hedging, and speculating needs in financial markets worldwide. Owing to standardized features, stock index futures contracts prosper and facilitate trading activities among different financial sectors. It is hence imperative for investors to be aware of co-varying degrees between cash and futures markets so as to gauge the hedging efficiency that results from futures positions and to calculate the hedging strategies to be carried forth. Theoretically, the minimum variance criterion proposed by Johnson (1960) is frequently invoked to derive the hedge ratio theoretically. Various econometric models, such as the OLS, ECM, and GARCH models, can be applied to estimate hedge ratios from empirical data for practical implementation.

This paper extends Engle's (2002) multivariate normal distribution based dynamic conditional correlation GARCH (DCC) model to a multivariate Student-*t* distribution based error correction dynamic conditional correlation GARCH (EC-DCC) model. The study herein tries to include the time-varying setting of correlation coefficients as well as the volatility on estimating optimal time-varying hedge ratios and hedge efficiency comparisons that are combined with the multivariate Student-*t* distribution-based estimation procedures. We empirically test the co-varying relationships among major stock and futures markets in the FTSE 100 and Nikkei 225 by considering the effect of the fat tail or leptokurtic characteristic of financial asset returns through the setting of the multivariate Student-*t* distribution-based error correction dynamic conditional correlation GARCH (EC-DCC) model instead of the normal distribution setting.

A minimum-variance hedge ratio is generally defined as the covariance of spot and futures prices divided by the variance of futures price, which can be estimated by the ordinary least square (OLS) regression. Other variables such as price spread, price change, and rate of return can be used interchangeably. For example, Hill and Schneeweis (1981) find that the minimum-variance hedge ratio is over-estimated by price variables in the British Pound and the Deutsche Mark currency markets. They suggest to use price spread variables instead in regression analyses in order to prevent problems resulting from series autocorrelation. Witt et al. (1987) present that the optimal crossing hedge ratios between wheat and corn markets should not be invariant after taking agents' different objective functions into account. Junkus and Lee (1985) compare hedge ratios based on four different objectives: risk eliminating, profit maximizing, risk minimizing, and utility maximizing. They use the Value Line, S&P500, and NYSE indexes and corresponding futures data to conduct an empirical study, showing the over-hedging results from traditional strategies and denoting the better performance of the variance-minimizing based hedge ratio.

Despite its simplicity, assumptions of the OLS regression do not coincide with characteristics of financial variables. For instance, the homoscedasticity condition requires the second moments of random variables to be constant, and the independent condition requires the error term variable to be not auto-correlated. An OLS based hedge ratio estimate naturally inherits the time-invariant and independent properties. However, the heteroscedasticity and self-correlation conditions are common in financial markets, and investors fail to adjust positions in their portfolio in time, because of an OLS based hedge ratio and an ill-performed portfolio risk management that is the result. On the other hand, the plain vanilla OLS regression model ignores long-term relationships among variables. Engle and Granger (1987) assert that two co-integrating series guarantee the existence of their error correction term containing abundant information. The short-run adjusting dynamics of the error correction term can help the achievement of equilibrium

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among variables in the long run. In addition, the non-stationary property of time series is visible in a financial market, including spot and futures price variables. Although the first-order difference operation can help satisfy the common stationary presumption in econometric analyses, it may eliminate long-term information contained inside variables.

Many studies in the literature adopt this idea by taking error correction terms into the OLS model so as to advance the estimation of hedge ratios and better the hedging efficiency. For instance, Ghosh (1993a) finds that the OLS model underestimates the hedge ratios of the S&P500 index futures contract on the S&P500, Dow Jones Industrial Average, and the NYSE composite indexes, because its ignores the co-integrating pattern. The error correction model can improve hedging performance, because of allowing for short-run adjusting dynamics and long-run equilibrium structure among variables. Their empirical results are consistent with Ghosh (1993b), Lien and Luo (1993), and Chou et al. (1996). While the error correction model resolves problems accompanied by ignoring the co-integration relationship among variables in contrast to the OLS model, both of them do not take the time-varying property of variances and correlations into account. Thus, the estimated hedge ratios are constant and incapable of conducting dynamic hedging strategies successfully.

The famous ARCH and GARCH models developed by Engle (1982) and Bollerslev (1986) remind us that the volatility clustering or heteroskedastic pattern is informative in predicting returns. They can be applied to advance the estimation of dynamic hedge ratios as well. In particular, a multivariate GARCH model (MGARCH) is suitable for hedge ratio estimation whenever portfolio risk management deals with positions more than one. For example, Bollerslev et al. (1988) first extend the univariate GARCH model to a VECH based MGARCH version which becomes the prototype for later MGARCH models. Bollerslev's (1990) constant conditional correlation MGARCH model (the CCC-MGARCH model) is widely-adopted in the literature for its great estimation

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simplicity. Engle and Kroner (1995) develop the BEKK-MGARCH model (named in terms of Baba, Engle, Kraft, and Kroner) to guarantee the positive semi-definite property which is neglected by the VECH based MGARCH model.

In terms of hedging effectiveness, Park and Switzer (1995) first apply MGARCH models on estimating the hedge ratios of S&P 500, MMI, and the Canadian Toronto 35 index futures contracts. They find that the CCC-MGARCH model performs better than the OLS and VAR models. Lypny and Powalla (1998) obtain similar empirical results in Germany's equity market. In contrast, Lien et al. (2002) find that the AR-CCC MGARCH model does not outperform the traditional OLS model in ten equity, currency, and commodity markets owing to the over-estimation of volatility persistence. Cochran et al. (2004) signify the sudden-change effect of volatility in four currency markets by the iterated cumulative sums of squares measure (the ICSS measure, Inclan and Tiao (1994)) and use it to complement CCC-MGARCH models.

Engle (2002) propose another new MGARCH model highlighting the time-varying property of dynamic conditional correlations (the DCC-MGARCH model), which is of particular importance for portfolio risk management though ignored by previous MGARCH models for estimation convenience. Therefore, we use the extended model to estimate optimal time-varying hedge ratios and conduct a hedging effectiveness comparison among the OLS, ECM, EC-CCC and EC-DCC models without and with the multivariate Student-*t* distribution based models in the FTSE 100 and Nikkei 225. We show that a futures contract can greatly reduce its corresponding cash variance and that the multivariate Student-*t* distribution based EC-DCC model performs best in both markets simultaneously. Therefore, we find that a new model developed by this study, based on Engle's (2002) DCC-MGARCH model with the error correction term in mean equation and multivariate Student-*t* distribution-based estimation procedure, outperforms the other models. This is because our empirical model takes all three important

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characteristics in the interaction between the stock and futures market into account: the long-term trend, the time-varying conditional correlation coefficients as well as volatility, and the multivariate Student-*t* distribution settings, i.e. the leptokurtic characteristic. The rest of this paper is organized as follows: Section II provides specifications of the models, Section III reports the empirical results, and Section IV concludes our findings.

II. Model Specifications

In a static sense, we know that due to the well-behaved mathematical properties of the normal distribution, it is the most visible setting in the literature. Nevertheless, financial time series data and assets returns have been well documented, and many alternatives or remedies have been proposed in the literature showing the fat tail or leptokurtic characteristic. The Student-*t* distribution setting is able to capture these characteristics, and it is often seen as an interesting and simple alternative to the normal distribution since it is characterized by a fat tail or leptokurtic characteristic. Dowd (1998) shows that Student-t distribution provides an easy way to capture uncertainty since it penalizes the lack of information regarding a portfolio's standard deviation with a wider VaR confidence interval. Kon (1984) and Hull and White (1998) assert that a discrete mixture of normal distributions can be used to explain the observed patterns of significant kurtosis and a positive skewness of daily data.

Hsieh (1989) estimates various forms of GARCH models based on a number of non-normal error densities for five foreign currencies. He finds that the Student-*t* and generalized error distributions perform better for the Canadian dollar and the Swiss franc. Jorion (1996) studies in the Value-at-Risk and provides a suggestion that the risk in Value-at-Risk itself should not be overlooked, although Value-at-Risk is an indispensable tool to control financial risks. The Student-*t* distribution is more consistent with financial reality than the normal distribution, because it endows extreme quantities with larger

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probabilities. He also concludes that the Student-*t* distribution setting is one way to remedy estimation errors instead of the normal distribution.

In the following we first develop a multivariate Student-*t* distribution based error correction DCC-MGARCH (EC-DCC) model to investigate dynamic interactions between stock and futures markets. This model estimates the optimal time-varying hedge ratios. A hedging efficiency comparison among our model and other usually used models is then conducted to shed light on the time-varying conditional correlation coefficients and the multivariate Student-*t* distribution settings.

In this paper we set the error correction terms in the mean equations for the returns in the spot and futures markets. In addition, we extend Engle's (2002) multivariate normal distribution based dynamic conditional correlation GARCH model to a multivariate Student-*t* distribution based dynamic conditional correlation GARCH model as the variance equation. The model setting in this paper is as follows.

The mean equations are:

$$s_t = \alpha_{0s} + \alpha_{1s}(S_{t-1} - \lambda F_{t-1}) + \varepsilon_{st}, \qquad (1)$$

$$f_t = \alpha_{0f} + \alpha_{1f}(S_{t-1} - \lambda F_{t-1}) + \varepsilon_{ft}, \qquad (2)$$

where $S_{t\text{-}1}$ presents the log prices of stock indices for FTSE 100 and Nikkei 225, and $F_{t\text{-}1}$ shows the log prices of stock index futures for FTSE 100 and the Nikkei 225, respectively; $S_{t-1} - \lambda F_{t-1}$ is the error correction term; and s_t and f_t are the log changes of spot index and futures index prices between time *t* and *t*-1, respectively. Our specification generalizes the conditionally normal basic structure to the Student-t conditional error distribution - that is, the conditional distribution of the innovation of normal distribution is replaced by Student-t distribution.

We now have the following setting:

$$\mathbf{u_t} | \Omega_{t-1} \sim f_{Student-t}(\mathbf{H_t}; \mathbf{u_t}; v) \qquad (3)$$

$$f_{Student-t}(\mathbf{H_t};\mathbf{u_t};v)=\frac{\Gamma[(n+v)/2]}{(\pi v)^{n/2}\Gamma(v/2)}\left|\mathbf{H_t}\right|^{-1/2}\left[1+v^{-1}\mathbf{u_t'}\mathbf{H_t^{-1}}\mathbf{u_t}\right]^{-(n+v)/2}, \quad (4)$$

where $\mathbf{u_t}=\begin{bmatrix}\varepsilon_{st}\\ \varepsilon_{ft}\end{bmatrix}$, ε_{st} and ε_{ft} are the residual terms; $H_t=\begin{bmatrix}h_{s,t} & h_{sf,t}\\ h_{sf,t} & h_{f,t}\end{bmatrix}$; H_t is the conditional variance-covariance matrix at time *t*, and its diagonal elements are h_{st} and h_{ft}, which are conditional variances of spot index and futures index returns, respectively; and v is the degree of freedom parameter. The variance equations are:

$$h_{s,t}=v_{0s}+v_{1s}\varepsilon_{s,t-1}{}^{2}+v_{2s}h_{s,t-1}, \quad (5)$$

$$h_{f,t}=v_{0f}+v_{1f}\varepsilon_{f,t-1}{}^{2}+v_{2f}h_{f,t-1}, \quad (6)$$

$$h_{sf,t}=\rho_{sf,t}\sqrt{h_{s,t}}\sqrt{h_{f,t}}, \quad (7)$$

$$\rho_{sf,t}=\frac{q_{sf,t}}{\sqrt{q_{ss,t}q_{ff,t}}}. \quad (8)$$

The key element of our interest in addition to conditional variances is the dynamic conditional correlations between the spot index and futures index returns, $\rho_{sf,t}$. Engle (2002) particularly structures the conditional correlation $q_{sf,t}$ in equation (8) as follows:

$$q_{sf,t}=\bar{\rho}_{sf}+\gamma(z_{s,t-1}z_{f,t-1}-\bar{\rho}_{sf})+\delta(q_{sf,t-1}-\bar{\rho}_{sf}), \quad (9)$$

where $\bar{\rho}_{sf}$, is constant unconditional correlation coefficient between stock index and stock index futures markets, and $z_{s,t}={\varepsilon_{st}}/{\sqrt{h_{s,t}}}$ and $z_{f,t}={\varepsilon_{ft}}/{\sqrt{h_{f,t}}}$ are the standardized residuals of the spot returns and of futures returns, respectively.[1] Note that the time-varying property of covariance $h_{sf,t}$ may result from the two time-varying standard deviations $\sqrt{h_{s,t}}$, and $\sqrt{h_{f,t}}$, or the time-varying correlation coefficient $\rho_{sf,t}$, because of $h_{sf,t}=\rho_{sf,t}\sqrt{h_{s,t}}\sqrt{h_{f,t}}$. However, the time-varying setting of $\rho_{sf,t}$ is often ignored by

1 The EC-CCC model contains all equations (1) through (9), except for the setting of the parameters of γ and δ which are the coefficients included in equation (9), i.e. $\gamma=\delta=0$.

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some econometric models for estimation simplification. Thus, we need to investigate the dynamics of the conditional correlation coefficients as well as volatility. Therefore, from the above setting, the optimal time-varying hedge ratios (b_t^*) can be calculated by

$$b_t^* = {}^{h_{sf,t}}\!/_{h_{f,t}} = \rho_{sf,t}\, {}^{\sqrt{h_{s,t}}}\!/_{\sqrt{h_{f,t}}}.$$

III Empirical Studies

This section uses the daily data of the FTSE 100 and the Nikkei 225 stock indices with their corresponding futures contracts listed in LIFFE and OSE during the period 1998/12/31–2006/12/29. Table 1 presents the descriptive and important statistics of the four series in terms of their rate of changes. It can be seen that the equity and futures markets in FTSE 100 and Nikkei 225 perform positively on average during the sampling period. Generally, the equity markets are less volatile than the futures markets, because of their smaller standard deviations. Normal distribution fittings for the four series are all rejected by the significant Jarque-Bera statistics and the kurtosis coefficients which are greater than 3. Thus, the leptokurtic or fat-tailed characteristic common in financial markets appears in our samples. This consolidates our setting by applying the multivariate Student-*t* distribution to replace the usual normal-distributed setting for our estimation procedures.

The heteroskedasticity or volatility clustering phenomenon prevails and justifies the implementation of MGARCH models according to the significant Ljung-Box Q statistics, i.e., the $Q^2(24)$ statistics. We also find that the existence of a unit root is strongly rejected for each series by the ADF test, and the two pairs of the spot and futures prices series are cointegrated significantly.[2] Consequently, the ECM setting can be applied to construct mean equations for MGARCH models.

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[2] The detailed test results for different specifications are not presented here for parsimony sake. They are available from the authors upon request.

— Insert Table 1 about here —

From the EC-DCC model which specified in section II, the model could be estimated by accounting for the dynamic conditional volatility and correlations as well as the long-run trend between stock and futures returns. Table 2 through Table 4 present estimation results of the EC-CCC and EC-DCC, both which are normal distribution based, and EC-DCC with the multivariate Student-*t* distribution based model, respectively. From these models' estimated results, we find that all parameters in the conditional variances are strongly significant. This result indicates that there is a very significant GARCH effect in the FTSE 100 and the Nikkei 225 stock indices and their corresponding futures contracts. From Table 3 and Table 4, we find that all parameters in the dynamic conditional correlation setting are strongly significant.

— Insert Table 2 about here —

— Insert Table 3 about here —

— Insert Table 4 about here —

It is obvious that the time-varying correlation coefficients between spot and futures positions are quite volatile from Figure 1 and Figure 2, which suggest the inadequacy of the constant correlation setting in the EC-CCC models and justifies the EC-DCC model. This result indicates that there is a very significant time-varying correlation effect between the spot stock index return and futures index return in both the FTSE 100 and the Nikkei 225 markets. We should not ignore this important effect in constructing econometric models, and of course in estimating the optimal time-varying hedge ratios. Finally, we see that the standardized residuals and squared standardized residuals present that there is no existing serial correlation between the spot and futures markets of FTSE 100 and the Nikkei 225 from Ljung-Box statistics. This result shows that there is very good fitting for these two markets from our model setting. This will make our estimate of the time-varying hedge ratios more efficacious.

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— Insert Figure 1 about here —

— Insert Figure 2 about here —

The above empirical results strongly bolster the argument that time-varying variances and correlation simultaneously exist in the stock and futures markets of FTSE 100 and Nikkei 225, which are two requirements for constructing the optimal dynamic hedge ratio. Thus, this paper adopts Ederington's (1979) framework to justify the time-varying property of the volatility structure along with the multivariate Student-*t* distribution setting and to measure our model's hedging effectiveness. Specifically, a stock index futures position is put into an unhedged spot stock index to construct a hedging portfolio. Ederington's (1979) original formula can be written as follows:

$$\sigma^2\left(s_t - b_t^* \ f_t\right), \tag{10}$$

where b_t^* is the optimal hedge ratio calculated by two static hedge ratios - the OLS and ECM - and three dynamic hedge ratios - the normal distribution based EC-CCC, the EC-DCC model, and the multivariate Student-*t* distribution based EC-DCC model, respectively; s_t denotes the spot return, and f_t is the futures price change rate. Figure 3 though Figure 4 show the optimal time-varying hedge ratio calculated by the estimations of the multivariate Student-*t* distribution based EC-DCC model in the FTSE 100 and the Nikkei 225, respectively.

— Insert Figure 3 about here —

— Insert Figure 4 about here —

Note that the optimal hedge ratios calculated by the OLS and ECM models are a static hedge ratio due to its constant over time result in contrast to the dynamic hedge ratio due to the time-varying ones calculated by the normal distribution based EC-CCC model, the EC-DCC model, and the multivariate Student-*t* distribution based EC-DCC models. The hedging effectiveness (*HE*) of a model can be defined as the reduced

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variance percentage that results from taking b_t^* units of a futures position into the unhedged portfolio - that is:

$$HE = \frac{\sigma_{\bullet}^{2}(unhedged) - \sigma_{\bullet}^{2}(hedged)}{\sigma_{\bullet}^{2}(unhedged)}. \quad (11)$$

A larger *HE* means a better hedging performance or a more precise forecasting ability on the future volatility structure among equity and futures positions in the hedged portfolio. Table 5 lists the *HE* results of different models. We find that the corresponding futures contracts can greatly reduce cash variances of the FTSE 100 and Nikkei 225 positions. It also can be seen that the multivariate Student-*t* distribution based EC-DCC model outperforms the other ones simultaneously in the two sampling groups, because it has the largest *HE* figures for the two sampling groups. Moreover, even with the normal distribution presumption, the EC-DCC model still performs better than the EC-CCC model and other static models. This may result from the fact that the CCC model fails to capture the time-varying dynamics of correlation coefficients. On the other hand, the EC-DCC model with the normal distribution setting performs worse than the EC-DCC model with the Student-*t* distribution setting in both scenarios. Consequently, the settings of the time-varying correlation coefficients and the multivariate Student-*t* distribution do matter and cannot be ignored especially for many fund managers conducting portfolios worth millions or billions.

— Insert Table 5 about here —

IV Conclusion

The volatility clustering or heteroskedasticity feature prevailing in financial markets underlies the dynamic property of optimal hedge ratios or covariances, which is neglected by the OLS and ECM models. On the other hand, the CCC-MGARCH models take the correlation component as a constant for estimation simplicity, which may fail to

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identify the fact that not only the variances but also the correlation coefficient matters in a time-varying covariance. In addition, the leptokurtic or fat-tailed characteristic is common in financial markets and have been proposed in the literature. Therefore, the simultaneous consideration of time-varying variances and correlation coefficients and a leptokurtic or fat-tailed characteristic is more consistent with reality and extraordinarily important for estimating optimal hedge ratios. This paper extends Engle's (2002) multivariate normal distribution based dynamic conditional correlation GARCH (DCC) model to a multivariate Student-*t* distribution based error correction dynamic conditional correlation GARCH (EC-DCC) model to investigate dynamic interactions between stock and futures markets. This model has estimated the optimal time-varying hedge ratios. A hedging efficiency comparison among our model and other usually used models has been conducted to shed light on the time-varying conditional correlation coefficients and the multivariate Student-*t* distribution settings. The empirical results of the hedging efficiency comparisons justify the above comments and denote the best performance of the multivariate Student-*t* distribution based EC-DCC model in the FTSE 100 and Nikkei 225.

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Table 1. Basic statistics of the samples' rate of changes

Data: The daily rates of change for the FTSE 100 and Nikkei 225 stock indices and their futures prices during the period 1998/12/31–2006/12/29 are used.

	FTSE 100		Nikkei 225	
	Spot	Futures	Spot	Futures
Number of Sample	2,086	2,086	2,086	2,086
Mean	0.675%	0.7%	2.625%	2.775%
Standard Deviation	281.175%	285.4%	338.075%	344.175%
Skewness	-0.2027***	-0.1619***	-0.1194***	-0.1194***
Kurtosis	6.10***	6.06***	4.88***	4.95***
Jarque-Bera	847.38***	821.66***	310.53***	336.71***
Q(24)	69.37***	70.22***	79.23***	82.18***
Q^2 (24)	2389.54***	2254.28***	322.43***	317.40***

1) *, **, and *** denote significance at the 10%, 5%, and 1% levels, separately.
2) The mean and standard deviation are calculated on a yearly basis.

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Table 2. The estimation results of the normal distribution based EC-CCC model

Data: The daily rates of change of the FTSE 100 and Nikkei 225 stock indices and their futures prices during the period 1998/12/31–2006/12/29 are used.

Parameters		FTSE 100		Nikkei 225	
Conditional mean					
α_{0s} ($\times10^{-3}$)		2.638	(4.138)***	5.669	(5.959)***
α_{1s} ($\times10^{-1}$)		-0.200	(-3.457)***	-4.985	(-5.473)***
α_{0f} ($\times10^{-2}$)		-1.490	(-22.224)***	-0.156	(-1.894)*
α_{1f}		0.138	(22.928)***	0.205	(2.201)**
Conditional variance					
v_{0s} ($\times10^{-6}$)		0.796	(3.527)***	3.853	(4.875)***
v_{0f} ($\times10^{-6}$)		0.761	(3.570)***	4.135	(4.920)***
v_{1s}		0.044	(8.042)***	0.063	(9.255)***
v_{1f}		0.043	(8.357)***	0.056	(8.867)***
v_{2s}		0.946	(131.849)***	0.915	(94.985)***
v_{2f}		0.948	(142.129)***	0.922	(98.229)***
Unconditional correlation					
ρ_{sf}		0.977	(117.987)***	0.973	(83.947)***
Spot	$Q(24)$	14.99		10.68	
	$Q^2(24)$	31.70		29.53	
Futures	$Q(24)$	18.38		11.65	
	$Q^2(24)$	28.08		26.80	

1) *, **, and *** denote significance at the 10%, 5%, and 1% levels, separately.
2) Figures in parentheses denote corresponding *t*-statistics.

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Table 3. The estimation results of the normal distribution based EC-DCC model

Data: The daily rates of change of the FTSE 100 and Nikkei 225 stock indices and their futures prices during the period 1998/12/31–2006/12/29 are used.

Parameters		FTSE 100		Nikkei 225	
Conditional mean					
α_{0s} ($\times 10^{-3}$)		2.407	(6.257)***	5.533	(6.162)***
α_{1s} ($\times 10^{-1}$)		-0.184	(-5.243)***	-4.969	(-5.764)***
α_{0f} ($\times 10^{-2}$)		-1.420	(-16.695))***	-0.177	(-1.927)*
α_{1f}		0.131	(17.149)***	0.220	(2.483)**
Conditional variance					
v_{0s} ($\times 10^{-6}$)		0.656	(3.818)***	2.525	(3.893)***
v_{0f} ($\times 10^{-6}$)		0.651	(3.700)***	2.924	(3.908)***
v_{1s}		0.051	(8.680)***	0.070	(7.998)***
v_{1f}		0.051	(8.760)***	0.063	(7.435)***
v_{2s}		0.944	(151.605)***	0.918	(90.867)***
v_{2f}		0.945	(153.509)***	0.922	(87.568)***
Conditional correlation					
γ		0.020	(5.617)***	0.031	(3.050)***
δ		0.974	(94.786)***	0.960	(56.392)***
Spot	$Q(24)$	14.42		10.18	
	$Q^2(24)$	29.63		26.01	
Futures	$Q(24)$	17.90		10.84	
	$Q^2(24)$	26.58		23.06	

1) *, **, and *** denote significance at the 10%, 5%, and 1% levels, separately.
2) Figures in parentheses denote corresponding *t*-statistics.

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Table 4. The estimation results of the multivariate Student-*t* distribution based EC-DCC model

Data: The daily rates of change of the FTSE 100 and Nikkei 225 stock indices and their futures prices during the period 1998/12/31–2006/12/29 are used.

Parameters		FTSE 100		Nikkei 225	
Conditional mean					
α_{0s} ($\times 10^{-3}$)		2.750	(4.591)***	5.545	(5.917)***
α_{1s} ($\times 10^{-1}$)		-0.214	(-3.921)***	-4.980	(-5.587)***
α_{0f} ($\times 10^{-2}$)		-1.380	(-18.245)***	-0.176	(-3.834)***
α_{1f}		0.128	(19.041)***	0.219	(3.392)***
Conditional variance					
v_{0s} ($\times 10^{-6}$)		0.667	(3.595)***	2.518	(3.739)***
v_{0f} ($\times 10^{-6}$)		0.661	(3.566)***	2.913	(3.829)***
v_{1s}		0.051	(8.567)***	0.070	(8.007)***
v_{1f}		0.051	(8.675)***	0.063	(7.415)***
v_{2s}		0.945	(146.961)***	0.918	(89.682)***
v_{2f}		0.947	(150.463)***	0.922	(87.257)***
Conditional correlation					
γ		0.019	(5.374)***	0.031	(3.149)***
δ		0.975	(84.312)***	0.960	(58.095)***
Spot	$Q(24)$	14.40		10.18	
	$Q^2(24)$	29.70		26.01	
Futures	$Q(24)$	17.93		10.85	
	$Q^2(24)$	26.84		23.04	

1) *, **, and *** denote significance at the 10%, 5%, and 1% levels, separately.
2) Figures in parentheses denote corresponding *t*-statistics.

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Table 5. Hedging effectiveness (HE) comparison of different models

Data: The daily rates of change of the FTSE 100 and Nikkei 225 stock indices and their futures prices during the period 1998/12/31–2006/12/29 are used.

Model	Average hedge ratio		Hedge effectiveness (*HE*)	
	FTSE 100	Nikkei 225	FTSE 100	Nikkei 225
OLS	0.986872	0.998970	88.8080%	82.7542%
ECM	0.9869	0.9990	89.9973%	83.8263%
EC-CCC model with normal distribution setting	0.967104	0.950789	91.2406%	85.8163%
EC-DCC model with normal distribution setting	0.969717	0.953604	93.0413%	91.5226%
EC-DCC model with Student-*t* distribution setting *	0.970138	0.953602	95.8573%	94.8293%

1) * denotes the best hedging efficiency in each hedged portfolio.

2) $$HE = \frac{\sigma_{\cdot}^{2}(unhedged) - \sigma_{\cdot}^{2}(hedged)}{\sigma_{\cdot}^{2}(unhedged)}$$

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Figure 1. Time-varying correlation coefficients of the samples' rates of change

Data: Sampling period covers the daily rates of change of the FTSE 100 stock indices and their futures prices during the period 1998/12/31–2006/12/29. The conditional correlation coefficient dynamics are calculated by the normal distribution based EC-CCC model and the multivariate Student-t distribution based EC-DCC model.

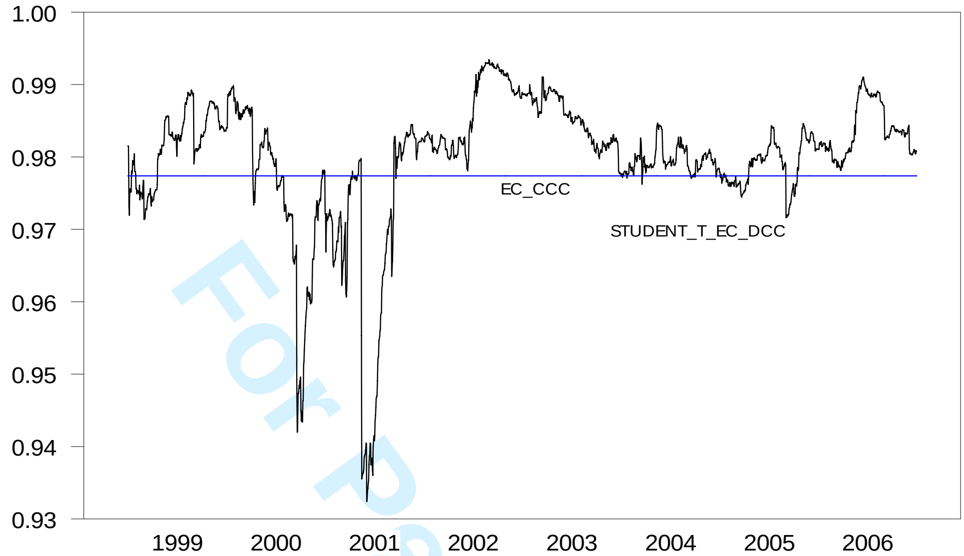

Figure 2. Time-varying correlation coefficients of the samples' rates of change

Data: Sampling period covers the daily rates of change of the Nikkei 225 stock indices and their futures prices during the period 1998/12/31–2006/12/29. The conditional correlation coefficient dynamics are calculated by the normal distribution based EC-CCC model and the multivariate Student-t distribution based EC-DCC model.

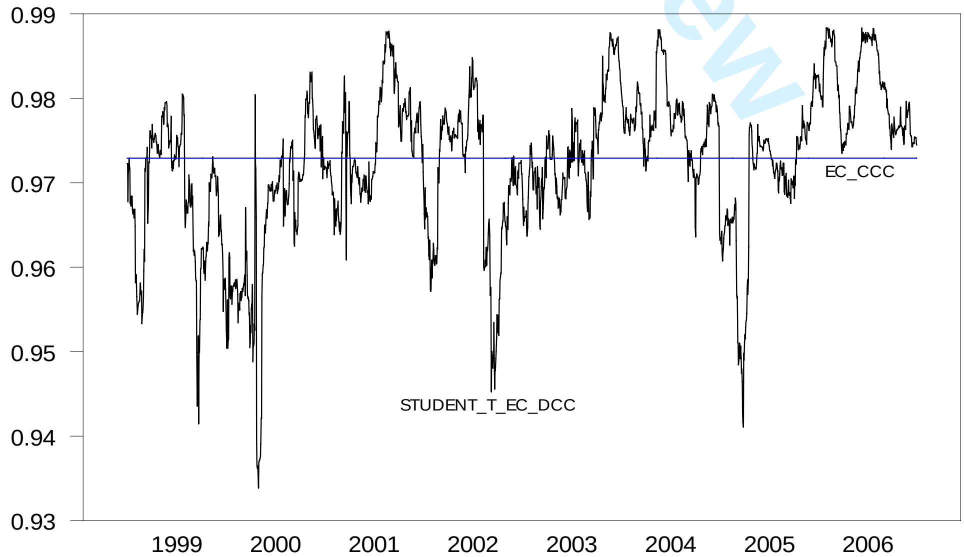

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Figure 3. The optimal time-varying hedge ratio calculated by the estimations of multivariate Student-t distribution based EC-DCC model for FTSE 100

Data: Sampling period covers the daily rates of change of the FTSE 100 stock indices and their futures prices during the period 1998/12/31–2006/12/29.

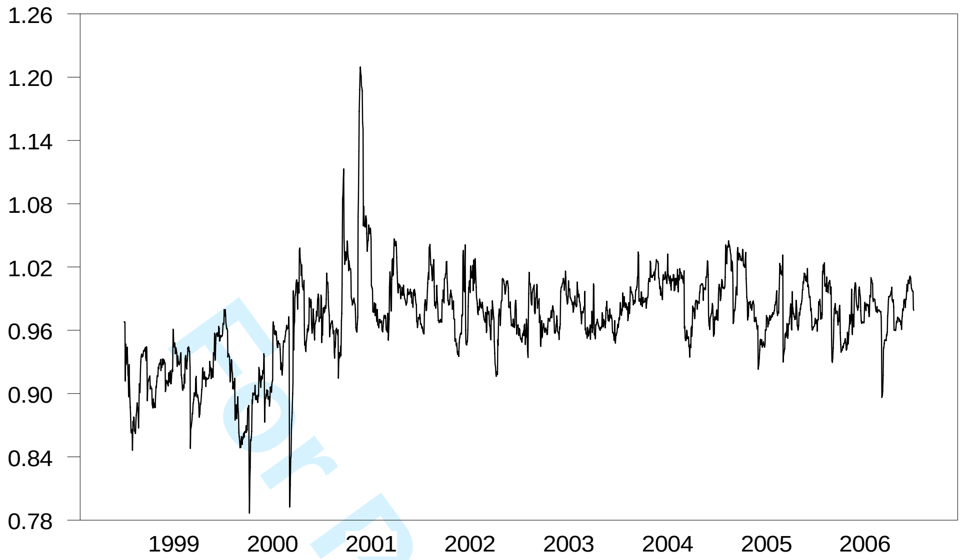
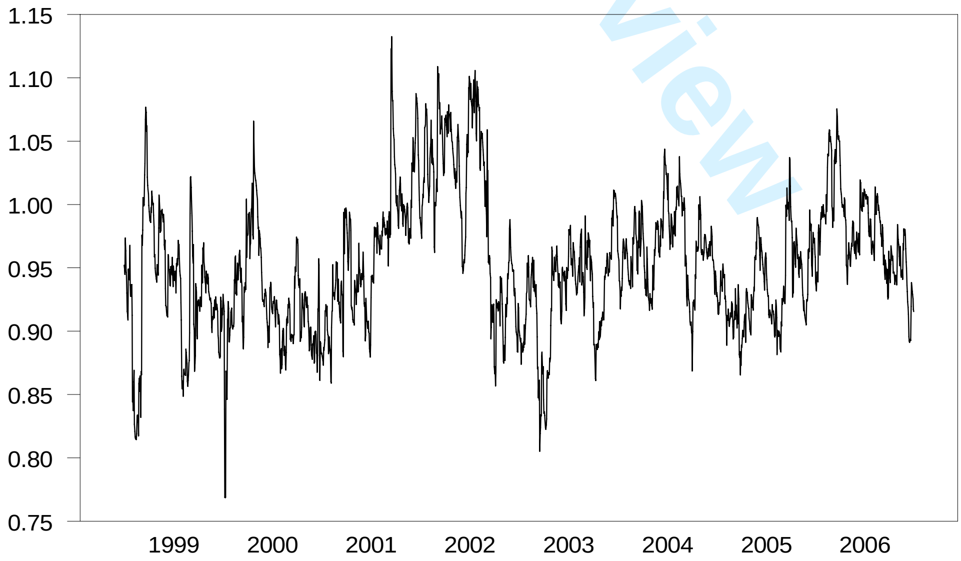


Figure 4. The optimal time-varying hedge ratio calculated by the estimations of multivariate Student-t distribution based EC-DCC model for Nikkei 225

Data: Sampling period covers the daily rates of change of the Nikkei 225 stock indices and their futures prices during the period 1998/12/31–2006/12/29.



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