Estimating portfolio value-at-risk via dynamic conditional correlation MGARCH modeliVan empirical study on foreign exchange rates

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1. Introduction

As the Basle Committee on Banking Supervision of the Bank for International Settlement begins to adopt Value at Risk (VaR) schemes in its capital accords, applications of VaR have begun to be popular and visible in academic studies and practical implementations. Econometric techniques such as GARCH models can be used in conjunction with VaR to better risk management because their notable capacities for depicting volatility-clustering phenomena as evident in financial dynamics (e.g. Bollerslev et al., 1992, Engle, 1995, Bollerslev, 2001, Li and Lin, 2004, and Cotter, 2005). In particular, co-varying relationships among positions in a portfolio are exceptionally essential because of risk diversification. Thus, we apply Engle's (2002) model which takes dynamic conditional correlation into consideration (the DCC model hereafter) to clarify this viewpoint. The rest of this study is structured as follows: Section 2 explores connections between VaR approaches and GARCH models; empirical results are demonstrated in Section 3; and Section 4 concludes our findings.

2. VaR and GARCH Models

We apply the variance-covariance approach advocated by JP Morgan's RiskMetrics (1996) to work out VaR figures, which can be stated as follows:

$$VaR_{t+1} = \begin{cases} -I_t\, C_\alpha \sqrt{\mathbf{w}_t{}'\, \boldsymbol{\Sigma}_t\, \mathbf{w}_t} & \text{for long positions;} \\ I_t\, C_{1-\alpha} \sqrt{\mathbf{w}_t{}'\, \boldsymbol{\Sigma}_t\, \mathbf{w}_t} & \text{for short positions.} \end{cases} \tag{1}$$

where I_t denotes an agent's invested amount at time *t*, and we set $I_t = 1$ without loss of generality. C_α and $C_{1-\alpha}$ are the left and right critical values, i.e. the risk multipliers of a presumed distribution with a given confidence level $1-\alpha$. JP Morgan uses the standard normal distribution and $\alpha=5\%$ for its risk management. This study follows its settings. $\mathbf{w}_t$ is the investment weights vector, i.e. $\mathbf{w}_t{}'=\left[w_1,\ w_2,...,w_n\right]$ where w_i denotes the percent invested in i-th asset in this portfolio. Σ_t is the estimated variance-covariance matrix of positions in this portfolio and can be estimated by various GARCH models. For example, Bollerslev et al. (1988) find that the conditional covariances are quite variable over time and are the significant determinant of time-varying risk premia for investors in bills, bonds, and stock markets. However, the traditional multivariate GARCH model involves estimation problems on the tradeoff between their generality and the number of parameters to be estimated. In addition, considerable restrictions are needed to guarantee the positive definiteness of the covariance matrix. Scholars have devoted work to overcoming these drawbacks, including Engle and Kroner (1995), Bollerslev (1990), and Engle (2002).

The BEKK model is suggested by Baba, Engle, Kraft and Kroner in a preliminary version of Engle and Kroner (1995). It guarantees the positive definiteness of the covariance matrix. Nevertheless, it is somewhat difficult to implement, and several strict constraints are required in its optimization iterations. Moreover, the BEKK model focuses on time-varying covariances, while the stochastic correlation coefficients are closer to reality (Longin and Solink, 1995, 2001).

In order to reduce the number of parameters to be estimated, Bollerslev (1990) develops a MGARCH model with constant conditional correlation presumption (the CCC model). That is, volatility of each individual series is estimated by a univariate GARCH estimation procedure in advance. Then the resulting standardized residuals are used to yield the conditional correlation matrix. In addition to the estimation flexibility, the CCC model endows the covariance matrix with the positive definite property simply by requiring each univariate conditional variance be positive and the constant matrix of conditional correlations be positive definite. Due to its computational simplicity, the CCC model is widely used in empirical applications. Nevertheless, a range of studies such as Tsui and Yu (1999), and Tse (2000) find that the assumption of constant conditional correlation can be too restrictive for realized data.

Later, Engle (2002) proposes a more generalized MGARCH model with dynamic conditional correlation setting (the DCC model). In contrast to the CCC and BEKK models, it simplifies the estimation procedures and allows time variation of the conditional correlation matrix. In terms of a portfolio, the DCC model first fits to each asset return an appropriate univariate GARCH model and estimates the conditional standard deviations to standardize the returns. Then the standardized return vector is used to model the correlation dynamics. The DCC model can be stated as follows:

$$\begin{aligned}
&\mathbf{r}_t = \boldsymbol{\mu}_t + \boldsymbol{\varepsilon}_t, \qquad \boldsymbol{\mu}_t = E\left[\mathbf{r}_t \middle| \Omega_{t-1}\right], \\
&\qquad\qquad\qquad\quad \boldsymbol{\varepsilon}_t \middle| \Omega_{t-1} \sim N\left(0, \mathbf{H}_t\right). \\
&\mathbf{H}_t = \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t, \quad \mathbf{D}_t = diag\left\{\sqrt{h_{ii,t}}\right\}. \\
&\mathbf{z}_t = \mathbf{D}_t^{-1} \boldsymbol{\varepsilon}_t
\end{aligned} \tag{2}$$

where $\mathbf{r}_t = \left(r_{1,t}, \cdots, r_{n,t}\right)'$, $\boldsymbol{\mu}_t = \left(\mu_{1,t}, \cdots, \mu_{n,t}\right)'$, and $\boldsymbol{\varepsilon}_t = \left(\varepsilon_{1,t}, \cdots, \varepsilon_{n,t}\right)'$ are vectors, $h_{ii,t}$ is the estimated conditional variance from individual univariate GARCH models, $\mathbf{D}_t$ is the diagonal matrix of conditional standard deviations, $\mathbf{R}_t$ is the time-varying conditional correlation matrix of returns, and $\mathbf{z}_t$ is the standardized residuals vector with mean zero and variance one. After the above basic construction, the dynamic correlation matrix of the DCC model can be specified further:

$$\begin{aligned}
\mathbf{R}_t &= \left(diag\left(\mathbf{Q}_t\right)^{-\frac{1}{2}} \right) \mathbf{Q}_t \left(diag\left(\mathbf{Q}_t\right)\right)^{-\frac{1}{2}} \\
\mathbf{Q}_t &= \left(q_{ij,t} \right) \\
\left(diag\left(\mathbf{Q}_t\right)\right)^{-\frac{1}{2}} &= diag\left(\frac{1}{\sqrt{q_{11,t}}}, \cdots, \frac{1}{\sqrt{q_{nn,t}}} \right) \\
q_{ij,t} &= \bar{\rho}_{ij} + \alpha\left(z_{i,t-1} z_{j,t-1} - \bar{\rho}_{ij} \right) + \beta\left(q_{ij,t-1} - \bar{\rho}_{ij} \right)
\end{aligned} \tag{3}$$

where $\bar{\rho}_{ij}$ is the unconditional correlations and the new time-varying conditional correlation coefficient $\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t}\, q_{jj,t}}}$. Engle (2002) suggests that the conditional correlation coefficients provide superior performance in a variety of situations.

3. Empirical Studies

In this section, we calculate VaR figures by various GARCH models. The data set includes the U.S. Dollar to British Pound (BP), Japanese Yen (YEN), and Euro Dollar (EU) exchange rates covering 1999–2004. Samples in period 1999–2003 are applied for parameter estimations, and samples in period 2004 are then used to calculate VaR figures under different MGARCH models for later comparisons.

Table 1 shows the basic daily rate-of-return statistics. From the significant ADF statistics, the null hypothesis of a unit root is rejected for all of the three series. Apparent leptokurtic or fat-tailed phenomenon appears in the foreign exchange markets because that the Kurtosis statistics are all larger than 3 and that the Jarque-Bera statistics are all significant. Moreover, obvious heteroskedastic pattern exists in that the Ljung-Box Q statistics are all significant. Thus, it is reasonable to apply GARCH models in the exchange markets.

— Insert Table 1 about here —

Intuitively, a large prediction failure number is uncomfortable in that it implies ill prediction ability or bad risk management. But an insensitive model may produce a smaller failure number at the price of larger deviation, which costs more resources to shelter risky assets. Thus, not only "the actual number of prediction failure" (NF for abbreviation hereafter), but also "the average deviation between VaR and realized return" (AD for abbreviation hereafter) is calculated. The specification of AD which can be used to measure the idle degrees of resources to shelter risk exposure is:

$$AD = \frac{1}{m}\sum_{t=1}^{m}\left(\left|VaR_t\right| - \left|r_t\right| \right)^{+} \tag{4}$$

where m is the number of trading days in the testing period, and the superscripted "plus" sign (+) out of the parentheses denotes that only the cases of effective risk management are taken into AD calculation, i.e. $\left|VaR_t\right| \geq \left|r_t\right|$. Perfect risk management requests low levels of NF and AD simultaneously.

However, no significant distinction of the EWMA, i.e. IGARCH (1, 1, $\lambda = 0.94$) and the GARCH (1, 1) models can be found on the criteria NF and AD for the three exchange rates data in Table 2. The two univariate GARCH models perform better and worse than each other at different criteria and different exchange rates data. For example, IGARCH has less prediction failures than GARCH in the BP case (33 < 41), but the opposite results exist in the EU and YEN cases (32 > 27 and 33 > 27). It may be the case that models from the univariate GARCH family exhibit equivalent ability to capture the volatility-clustering dynamics of exchange markets in terms of VaR backtesting.

— Insert Table 2 about here —

Next, the econometric structures and parameter estimates of the DCC, CCC, and BEKK MGARCH models are listed in Table 3. We then use the three exchange rates to formulate a hypothetical equally-weighted portfolio to examine the performance of the three MGARCH models to clarify whether the time-varying correlation setting is important for portfolio risk management. According to Table 4, none of the three models performs best on the two criteria simultaneously. The BEKK model performs best at the AD criterion and the DCC model performs best at the NF criterion. Nevertheless, their AD figures are rather close because each of them differs less than 2.11% $\left(=\left(0.4244-0.4157\right)/\,0.4157\right)$ with each other. In contrast, the NF criterion reveals a strong ranking order in which the DCC model performs notably 42.11% $\left(=\left(27-19\right)/\,19\right)$ better than the CCC model and notably 57.89% $\left(=\left(30-19\right)/\,19\right)$ better than the BEKK model. In other words, the DCC model seems to be the best choice for portfolio VaR calculation because it offers great improvement of prediction accuracy at the lesser cost of sheltering resource. We can reasonably infer that the superiority of the DCC model results from the consideration of time-varying correlation among different exchange rate series, which is more consistent with reality.

— Insert Table 3 about here —

— Insert Table 4 about here —

4. Conclusions

This study uses exchange rates data to examine performance of GARCH models in terms of VaR backtesting on prediction failures and average deviations. We find that univariate GARCH models seem to be equipped with equivalent ability to capture the volatility-clustering dynamics of financial assets. In contrast, the multivariate GARCH model developed by Engle (2002) performs much better on the criterion of prediction failures than other multivariate GARCH models developed by Bollerslev (1990) and Engle and Kroner (1995) in estimating portfolio VaR series. Consequently, it is reasonable to infer that the superiority results from the consideration of time-varying correlation and cannot be ignored when dealing with portfolio risk management.

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Table 1 Basic Statistics of Three Series

of Daily Proportional Changes in Exchange Rates

	BP	EU	YEN
	Basic Statistics		
Number of sample	1249	1249	1249
Mean	-0.0061	-0.0054	-0.003
Maximum	1.504	2.3842	4.9073
Minimum	-2.006	-2.3177	-3.4033
Variance	0.2409	0.4375	0.4119
Standard deviation	0.4908	0.6614	0.6418
Skewness	-0.0039	-0.022	0.2291
Kurtosis	3.6369	3.6058	6.8525
	Test Statistics		
Jarque-Bera	21.7322*	19.5668*	792.5033*
ADF	-15.0679*	-14.6651*	-15.223*
Q(20)	12.815	19.735	11.282
$Q^2(20)$	46.949*	53.877*	48.811*

* denotes significance at the 5% level.

Table 2 Numbers of Prediction Failure and Levels of Average Deviations for Year 2004 — EWMA vs. GARCH VaR

	EWMA			
	Numbers of Prediction Failure (NF)			Average Deviations (AD%)
	Long positions	Short Positions	Total	
BP	14	19	33	0.6167
EU	17	15	32	0.6915
YEN	18	15	33	0.5972
	GARCH			
	Numbers of Prediction Failure (NF)			Average Deviations (AD%)
	Long positions	Short Positions	Total	
BP	19	22	41	0.5346
EU	15	12	27	0.6928
YEN	14	13	27	0.6299

Table 3 Econometric structures and parameter estimates of DCC, CCC, and BEKK MGARCH Models

The unified mean equation of the three models:

$$r_{i,t}=\mu_i+\varepsilon_{i,t},\quad \boldsymbol{\varepsilon}_t|\Omega_{t-1}\sim N(0,H_t),\quad H_t\equiv\begin{bmatrix} h_{1,t} & h_{12,t} & h_{13,t} \\ h_{21,t} & h_{2,t} & h_{23,t} \\ h_{31,t} & h_{32,t} & h_{3,t} \end{bmatrix}.$$

The variance equation of the DCC model ($i,j=1\equiv$ EU, $i,j=2\equiv$ YEN, $i,j=3\equiv$ BP):

$$h_{1,t}=C_{11}+a_{11}\varepsilon^2_{1,t-1}+b_{11}h_{1,t-1}\qquad h_{2,t}=C_{22}+a_{22}\varepsilon^2_{2,t-1}+b_{22}h_{2,t-1}$$

$$h_{3,t}=C_{33}+a_{33}\varepsilon^2_{3,t-1}+b_{33}h_{3,t-1}\qquad q_{ij,t}=\bar{\rho}_{ij}+\alpha(z_{i,t-1}z_{j,t-1}-\bar{\rho}_{ij})+\beta(q_{ij,t-1}-\bar{\rho}_{ij})$$

$$\rho_{ij,t}=\frac{q_{ij,t}}{\sqrt{q_{ii,t}\,q_{jj,t}}}\qquad h_{ij,t}=\rho_{ij,t}\sqrt{h_{i,t}}\sqrt{h_{j,t}}$$

The variance equation of the CCC model ($i,j=1\equiv$ EU, $i,j=2\equiv$ YEN, $i,j=3\equiv$ BP):

$h_{i,t}=C_{ii}+a_{ii}\varepsilon^2_{i,t-1}+b_{ii}h_{i,t}$, $h_{ij,t}=\rho_{ij}\sqrt{h_{i,t}}\sqrt{h_{j,t}}$, where ρ_{ij} is constant unconditional correlation.

The variance equation of the BEKK model ($i,j=1\equiv$ EU, $i,j=2\equiv$ YEN, $i,j=3\equiv$ BP):

$$H_t=\begin{pmatrix} C_{11} & C_{12} & C_{13} \\ 0 & C_{22} & C_{23} \\ 0 & 0 & C_{33} \end{pmatrix}'\begin{pmatrix} C_{11} & C_{12} & C_{13} \\ 0 & C_{22} & C_{23} \\ 0 & 0 & C_{33} \end{pmatrix}+\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}'H_{t-1}\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$$

$$+\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}'\begin{pmatrix} \varepsilon^2_{1,t-1} & \varepsilon_{1,t-1}\varepsilon_{2,t-1} & \varepsilon_{1,t-1}\varepsilon_{3,t-1} \\ \varepsilon_{2,t-1}\varepsilon_{1,t-1} & \varepsilon^2_{2,t-1} & \varepsilon_{2,t-1}\varepsilon_{3,t-1} \\ \varepsilon_{3,t-1}\varepsilon_{1,t-1} & \varepsilon_{3,t-1}\varepsilon_{2,t-1} & \varepsilon^2_{3,t-1} \end{pmatrix}\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

	DCC		CCC		BEKK	
Mean equation						
μ_1	0.02765907	(2.25023)**	-0.0299103	(-2.15303)**	-0.0097799	(-0.6581)
μ_2	0.00953727	(2.71413)***	-0.0010481	(-0.07172)	-0.0002405	(-0.01557)
μ_3	0.01259293	(2.83825)***	-0.0295238	(-2.95291)***	-0.0112213	(-0.97876)
Conditional variance						
C_{11}	0.01606457	(192.4624)***	0.17600768	(9.02761)***	0.09229813	(3.83916)***
C_{12}	-		-		0.07781377	(2.23269)**
C_{13}	-		-		0.04356409	(1.86766)*
C_{22}	0.03058556	(5.73953)***	0.02570124	(4.63947)***	0.10208315	(2.66283)***
C_{23}	-		-		0.00123405	(0.05179)
C_{33}	0.01168708	(39.8515)***	0.02216984	(24.3383)***	0.04081853	(2.53676)**
a_{11}	0.00631243	(3.59173)***	0.02806241	(3.73930)***	-0.1150398	(-4.46604)***
a_{12}	-		-		-0.1223074	(-3.39018)***
a_{13}	-		-		0.06154895	(3.02410)***
a_{21}	-		-		-0.0429973	(-1.82635)*
a_{22}	0.06889311	(5.20071)***	0.09400469	(11.9998)***	-0.1999086	(-6.77908)***
a_{23}	-		-		-0.0115973	(-0.67717)
a_{31}	-		-		-0.0528492	(-1.44721)
a_{32}	-		-		0.14470230	(3.50212)***
a_{33}	0.03410842	(9.51350)***	0.04728167	(16.2726)***	-0.1964761	(-7.11189)***
b_{11}	0.95997063	(9.8556)***	0.58474087	(14.6191)***	0.99576367	(139.0897)***
b_{12}	-		-		-0.02993147	(-2.71481)***

(*Continued on next page*)

	DCC		CCC		BEKK	
b_{13}	-		-		0.02395819	(4.39177)***
b_{21}	-		-		-0.02027279	(-2.01272)**
b_{22}	0.85821571	(36.7895)***	0.85042075	(83.4550)***	0.95217287	(76.19464)***
b_{23}	-		-		-0.01416988	(-2.20495)**
b_{31}	-		-		-0.03306931	(-2.81989)***
b_{32}	-		-		0.04528157	(3.29903)***
b_{33}	0.92228510	(24.5844)***	0.87422945	(146.966)***	0.96183429	(128.9099)***
Conditional correlation						
α	0.03399787	(4.59058)***	-		-	
β	0.91940272	(39.7378)***	-		-	
Unconditional correlation						
ρ_{12}	-		0.11251262	(5.53328)***	-	
ρ_{13}	-		0.65578882	(61.2153)***	-	
ρ_{23}	-		0.11648389	(6.07363)***	-	

***, **, and * represent significance at the 1%, 5%, and 10% levels, respectively. The parameters of α and β are the coefficients of dynamic conditional correlation. The CCC-MGARCH model contains all equations in DCC-MGARCH, except for $\alpha = \beta = 0$. Figures in parentheses denote corresponding *t*-statistics.

Table 4 Numbers of Prediction Failures and Levels of Average Deviations for Year 2004 — CCC vs. BEKK vs. DCC VaR

	Numbers of Prediction Failure (NF)			Average
	Long positions	Short Positions	Total	Deviations (AD%)
DCC	**10**	**9**	**19**	**0.4244**
CCC	15	12	27	0.4218
BEKK	16	14	30	0.4157