



Discrete Optimization

The design of reverse distribution networks: Models and solution procedures

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Abstract

Reverse distribution, or the management of product return flows, induced by various forms of reuse of products and materials, has received growing attention throughout this decade. In this paper we discuss reverse distribution, and propose a mathematical programming model for a version of this problem. Due to the complexity of the proposed model, we introduce a heuristic solution methodology for this problem. The solution methodology complements a heuristic concentration procedure, where sub-problems with reduced sets of decision variables are iteratively solved to optimality. Based on the solutions from the sub-problems, a final concentration set of potential facility sites is constructed, and this problem is solved to optimality. The potential facility sites are then expanded in a greedy fashion to obtain the final solution. This “heuristic expansion” was also performed using the solution found with a greedy heuristic to provide a short-list of potential facility sites. Computational tests demonstrate a great deal of promise for this solution method, as high-quality solutions are obtained while expending modest computational effort.

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1. Introduction

Reverse logistics encompasses the logistics activities all the way from used products no longer required by the user to products again usable in the market. It is the process of planning, implementing, and controlling the efficient, cost effective flow of raw materials, in-process inventory, finished goods and related information from the

point of consumption to the point of origin for the purpose of recapturing value or proper disposal (Stock, 1998). The most intuitively related notion with such reverse activities involves the physical transportation of used products from the end user back to the producer. Reverse distribution activities involve the removal of defective and environmentally hazardous products from the hands of customers. This also includes products that have reached the end of their usable life. It is a process whereby companies can become more environmentally efficient through reusing and reducing the amount of materials used.

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In the modern society, we invest a great deal in the manufacturing of consumer durable goods. Manufacturing in economic quantities requires significant investment in terms of capital, labor, energy and raw materials. Unfortunately, all products have a finite life expectancy. When original goods reach the end of their useful life they are typically discarded or otherwise disposed of by their owners. As more and more companies are becoming environmentally conscious, and as stringent environmental laws are being passed, goods that breakdown or reach the end of their usable life are being recalled or repossessed by the manufacturer. In many cases, the original manufacturer is refurbishing these products. An example is Hewlett-Packard, who collects empty laser-printer cartridges from the consumers for reuse.

This reverse distribution activity can be crucial to the survival of companies, because the permanent goodwill of the company is at stake. Businesses succeed because they respond to both external and internal changes and adjust in an effective manner to remain competitive. To achieve its business objectives, a company must respond to increasing customer demand for “green” products, comply with strict environmental regulations, and implement environmentally responsible plans as a good corporate citizen. For example, Church and Dwight Co. Inc, the owner of Arm and Hammer, estimates that the loyalty of customers who appreciate the company’s clean-and-green image translates into 5–15% more revenues per year (or about \$75 million) (Ottoman, 1998). Moreover, corporations are becoming more and more sensitive to the needs of the natural environment. There is a tremendous pressure from corporate stakeholders, the community, employees, government and customers for organizations to be environmentally conscious. Organizations have also realized that maintaining a public image of being environmentally conscious could carry over to their suppliers and the supply chain in such a manner that they also are evaluated on their own environmental performance.

There are a growing number of firms interested in minimizing the environmental impact of their products and services, and an increasing interest in taking a proactive rather than an “end-of-pipe-

line” approach (Beckman et al., 1995; Thomas and Griffin, 1996). Many firms are becoming aware that clean products and processes produce less waste for disposal and this reduces the potential for liability. In the past 10 years, Germany has passed two legislative acts, the packaging law and consumer electronics act, which make the manufacturer responsible for (1) reuse of packaging materials and (2) responsibility for the end-of-life disposition of all consumer electronic goods (Rousoo and Shah, 1994). The strict environmental laws passed in Germany and other European countries also state that consumers have the right to leave packaging materials at retail store outlets, and that stores must dispose them properly or return them to their manufacturer to be remanufactured and reused. Norway and Denmark, for example, have for many years required beverage containers to be reusable. The intent is to minimize the amount of materials being land filled, and minimize the waste of recoverable resources. These acts require that firms must address the reverse flow of items from the consumer.

This paper makes two primary contributions. First it proposes strong and weak mathematical programming formulations for a reverse distribution problem. Second, new solution methods are developed for this problem. In the next section we discuss the elements of reverse logistics, and provide a literature review on reverse logistics systems. Section 3 develops the reverse distribution model and problem formulations. Section 4 develops the heuristic solution procedures used to solve the problem. Section 5 contains the computational results and analysis, whereas summary and conclusions are provided in Section 6 of the paper.

2. Reverse logistics

Reverse logistics deals with four basic tenets: reduce, substitution, reuse and recycle. The operations of reverse channels of distribution are receiving increased attention as rampant solid waste pollution, frequent energy shortages, and serious materials scarcity are recognized as realities of our modern age. If companies are to survive in this modern era, they will have to plan, organize, and

manage specifically for environmental, energy, and materials contingencies.

There are two major supply chains to be concerned within any distribution system: the forward chain, and the reverse supply chain (reverse logistics system). The forward chain is a well-researched topic where the strategy is to distribute the products from manufacturing plants or factories to customer outlet zones. There has been a lot of work published in this area. The capacitated plant location problem (CPLP) is one of the well-researched problems in this area (Davis and Ray, 1969; Geoffrion and Graves, 1974; Guignard and Spielberg, 1979; Barcelo and Casanovas, 1984; Lee, 1993; Tragantalerngsak et al., 1997). For an overview of research on the plant location problem, the interested reader is referred to survey articles in this area (Krarup and Pruzan, 1983; Sridharan, 1995; ReVelle and Laporte, 1996).

The reverse chain is when a product or component returns to the production chain after its use, either for purposes of repair, recycling, or remanufacturing. Recovery of used products has become a field of rapidly growing importance. A number of papers have been published recently on the issue of product recovery network (Bloemhof-Ruwaard et al., 1999; Fleischmann et al., 2000a,b; Krikke et al., 1999a). These papers include product types such as carpeting (Louwers et al., 1999), copiers (Krikke et al., 1999b), steel by-products (Spengler et al., 1997), reusable packaging (Kroon and Vrijens, 1995) and sand (Barros et al., 1998).

Sarkis et al. (1995) provided three important characteristics that differentiates a reverse logistics system from a traditional supply chain system:

1. Most logistics systems are not equipped to handle product movement in a reverse channel;
2. Reverse distribution costs may be higher than moving the original product from the manufacturing site to the consumer;
3. Returned goods often cannot be transported, stored, or handled in the same manner as in the regular channel.

Fleischmann et al. (1997) point out that reverse distribution is not necessarily a symmetric picture of forward distribution. Schuldenfrei and Shapiro

(1980) believed that the pressures of inflation, tight energy suppliers, and the trend of higher costs of logistics would force management to look at physical collection that is just as important as physical distribution.

2.1. *Product recalls and refurbishing*

In this paper we look at an important component of environmentally conscious manufacturing: product recalls (or returns). Product recall is a reverse distribution activity that withdraws goods from consumers. The products are either hazardous (e.g. motor oil), defective, or have reached the end of their useful life (e.g. printer cartridges). In particular, this paper examines product recall situations in which the customer returns the product to a retail store and the product is sent to a refurbishing site which will rework the product or dispose it properly. Costs of product recall through the reverse distribution channel are at least two or three times higher than costs incurred in forward distribution, often due to small quantities of shipments, fluctuating and uncertain demand, and the urgency involved in the recall process (Chandran and Lancioni, 1981; Min, 1989).

A refurbishing facility transforms the returned product into units that satisfy exactly the same quality and other standards as new units (Lund, 1984). The remanufactured product provides the consumer a product with a value not otherwise available, the retailer some additional business, the remanufacturer additional work, and society a reduction in the drain on physical resources. Corporations are introducing strategies related to the many “R’s” of environmental issues. Included among them are recycling, reduction, reclamation, recovery, reuse and remanufacturing (Guintini and Andel, 1995). Ever since the US Congress passed the Consumer Product Safety Act of 1972, the Consumer Product Safety Commission (CPSC) has the power to order a manufacturer to notify the public and recall for repair, replacement, refund, or destruction any product that poses a hazard for the environment. In 1988, the US CPSC was involved in some 221 recalls of defective products, covering about 8 million units. Five

years later, in 1993, this number had risen to 367 recalls covering about 28 million product units (Smith et al., 1996).

In a study by Monczka and Trent (1995), purchasing and materials managers' second highest rated future concern was the impact of environmental regulation on business activities. Recalls for defective, new, and established products occur often, and they can have serious repercussions. In several cases, they threatened to, and actually did, destroy brands and even companies. However, companies still are not prepared to deal with recalls nor do they recognize how great an impact a recall can have on an organization's reputation. The point is that the risks are higher than ever for the company that fails to anticipate the possibility of a product recall.

Contingency planning for product recall involves nearly every function within a business. Attempting to recall products that cannot be easily located within the distribution system can be a very expensive operation. The final product recall decision concerns how goods are to be moved back through distribution channel, or reverse distribution system design. Designing the channel for return movement requires consideration of the product characteristics, customer and demand characteristics, as well as the nature of the current distribution system.

One of the first studies that investigated product recall was done by Fisk and Chandran (1975) who looked at traceability mechanisms for both durable and non-durable goods in an attempt to efficiently track the hazardous and defective products. However, the study did not develop any reverse distribution strategies. Murphy (1986) conducted an empirical study to obtain information concerning transportation and warehousing issues in a product recall procedure. The study did not provide any model that would minimize reverse distribution costs nor did it develop any reverse distribution strategies. Min (1989) developed a goal-programming model that focused on choosing transportation modes that minimized transportation and reverse distribution costs against shipping times in product recall situations. However, it was limited in solving small-sized problems using an off-the-shelf commercial software.

2.2. *Other relevant literature*

Caruso et al. (1993) developed a location-allocation model for planning urban solid waste management systems. The results of the model are the number and location of waste disposal plants, specifying the technology adopted, the amount of waste processed and the service basin of each plant. They applied their problem and heuristic procedure to the case of the Italian region Lombardy for regional management of waste disposal. Bloemhof-Ruwaard et al. (1994) studied the problem of coordinating product and by-product flows in a two-level distribution network. The model formulation and solution procedures suggested in the paper were the first attempt at studying the coordinated control of product and by-product flows within distribution networks. The international nature and complexity of many environmental problems makes it almost impossible to make decisions based on intuition. A quantitative model-based representation of the problem will often be extremely useful (Bloemhof-Ruwaard et al., 1995). More recently Carter and Ellram (1998) reviewed the literature on reverse logistics and suggested some critical factors in the reverse logistics process. Specifically, Carter and Ellram proposed that sincere shareholder commitment and top management support are necessary for the continued success of a reverse logistics program.

It is important to point out that in practice, products are also returned by the existing distribution channels (e.g., packaging material, copier machines). Fleischmann et al. (2000a) investigated the question whether to integrate collection and recovery with the original distribution network or separate both channels. They conclude that the influence of product recovery is context dependent and point to a need to look at a comprehensive approach to redesigning a company's logistics network in an integral way for cases where existing distribution channels could no longer be used. From a methodological standpoint, forward and return networks can be modeled separately in many cases. This would lead to significant reduction in problem sizes. Our research addresses this latter situation.

In Section 3 of this paper we develop an analytical model that minimizes reverse distribution costs. This model builds upon the single-source plant location model developed by Pirkul and Jayaraman (1996) that restricted supply of customer demand from a single distribution center. However, our model does not restrict supply of reverse distribution products from a single origination site (i.e. flow may be split). Taking a coordination cost view, it is reasonable that information technology now facilitates these previously unreasonable assumptions (Clemons et al., 1993). Further we ensure a tight bound on the number of collection sites and refurbishing sites that can be open. Addition of these constraints increases the complexity level of the mathematical programming model developed in this paper.

Although on the surface it may seem the best strategy to recall products from distributors and customers through existing distribution channels, this may not be wise (Ballou, 1992). One danger is the possibility of contamination of the good product flowing in the channel with the recalled product (for example, tainted off-the-shelf pain medicine versus good product). In such cases, the recalling firm may establish a separate channel (public warehousing and for-hire trucking, for example) to specifically handle the recall. Traditionally, goods were considered to flow from manufacturer to consumers through retail outlets. Customer service reflected the idea of supplying a customer, not servicing a customer. Now, however, electronic commerce, the consumerism movement as well as the recycling movement, have generated concern for customer service after the sale of the product. Thus, the logistician must be concerned with designing product flow channels to satisfy customer needs after purchase as well. It is critical that manufacturers must develop effective recall strategies, not only to comply with CPSC (or similar) requirements, but also to limit injuries to customers, avoid costly legal battles, and to keep the company, product line, and brand viable.

In this study, we assume that the product has already traveled through the forward distribution phase and is currently in the hands of the customers. Customers who are now stuck with a hazardous or a defective product that is harmful to

the environment, or a product that has reached the end of its usable life, have the option to return the product to their closest origination site. This origination site would serve as a collection point, where customers also receive resolution if necessary (refund, new product, refurbished product, etc.).

3. The reverse distribution model

In order to formulate a mathematical model for the reverse distribution problem, we make the following assumptions:

1. The hazardous products are currently located at source (retail) outlets (origination sites). The model that is proposed in this paper would find an efficient strategy to return the defective products from a set of origination sites to specific collection sites, which in turn will ship them to refurbishing sites for remanufacturing/proper disposal.
2. The retailer/wholesaler is considered to be an initial collection point. This is a realistic assumption because the customer would be inclined to return the product to the closest origination site to get a refund or to purchase another product. In some cases such as hospitals and oil-change shops, the retailer generates hazardous waste product at the retail site itself. The hazardous products may be shipped directly to the refurbishing site at a substantially higher variable cost.
3. There is a fixed cost to opening collection sites and refurbishing sites. There is a limit to the number of collection sites and refurbishing sites that can be opened, but the choice of which collection sites and which refurbishing sites to be opened must be decided by the model. Shipment directly from an origination site to a refurbishing site is allowed, but the variable cost is much higher than shipping through a collection site. One reason for this is that small lot sizes cost more to ship. For example, nuclear and biomedical waste from hospitals may only be allowed to be temporarily collected (prior to shipment to the refurbishing/disposal site) at a limited number of hospitals in the region. If

direct shipment is not allowed, then assigning an infinite cost to opening the direct shipment “site” will prohibit this.

Even with these assumptions, it is important to note that the model proposed below can also be adapted for other type of products (end-of-life, commercial returns) and other reverse functions (recycling, remanufacturing, reuse, refurbishing).

The following notation will be used to describe the model:

- Product: This includes products that:
 - have been recalled,
 - are to be recycled,
 - are to be disposed, or
 - are hazardous.
- I — $\{i/i$ is an origination site $\}$. This is a store, a retail outlet, or a customer collection station. All products are received from customers at origination sites and are passed to collection sites.
- J — $\{j/j$ is a collection site $\}$. Collection sites are synonymous with intermediate transshipment sites. A collection site receives the collected products from the origination sites. The last collection site denotes a direct shipment from the origination site to the refurbishing facility site at a premium variable cost, thus preventing infeasibility. Note that no product originates at any collection site.
- K — $\{k/k$ is a refurbishing facility site $\}$. This site is:
 - a refurbishing site,
 - a recycling plant,
 - a decontamination plant, or
 - the original manufacturing site.

The last refurbishing facility site is a dummy site with infinite cost and infinite capacity, and prevents infeasibility in the solution procedure due to insufficient capacity.
- C_{ijk} —Total variable cost of transporting a single unit of recalled product from origination site i through collection site j and onto refurbishing site k .

This includes the per unit costs for:

- Processing the recalled product at the origination site.

- The inbound and outbound transportation costs for sending the recalled products from the origination sites to refurbishing sites via the collection sites.

- F_j —Cost of opening a collection site j .
- G_k —Cost of opening a refurbishing site k .
- a_i —Number of hazardous products residing at origination site i .
- B_j —Maximum capacity of collection site j .
- D_k —Maximum capacity of refurbishing facility k .
- P_{\min} —Minimum number of collection sites to open and operate.
- P_{\max} —Maximum number of collection sites to open and operate.
- Q_{\min} —Minimum number of refurbishing facilities to operate.
- Q_{\max} —Maximum number of refurbishing facilities to operate. Note that B_0 is set to some arbitrarily high value (999,999).

The decision variables for the model are:

- X_{ijk} —fraction of units at origination site i that is transported through collection site j and onto refurbishing site k . Use of the index $j = 0$ indicates that the fractional demand is assigned directly from i to k . The index value zero (0) is not used for subscripts i and k .
- $P_j = \begin{cases} 1 & \text{if collection site } j \text{ is open,} \\ 0 & \text{otherwise.} \end{cases}$
- $Q_k = \begin{cases} 1 & \text{if refurbishing facility } k \text{ is open,} \\ 0 & \text{otherwise.} \end{cases}$

A strong formulation of this problem can now be stated as:

3.1. Model refurb

$$\begin{aligned} \text{Min } Z = & \sum_i \sum_j \sum_k C_{ijk} a_i X_{ijk} + \sum_j F_j P_j \\ & + \sum_k G_k Q_k \end{aligned}$$

subject to:

$$\sum_j \sum_k X_{ijk} = 1 \quad \text{for all } i, \tag{1}$$

$$\sum_i \sum_k a_i X_{ijk} \leq B_j \quad \text{for all } j, \tag{2}$$

$$\sum_i \sum_j a_{ij} X_{ijk} \leq D_k \quad \text{for all } k, \tag{3}$$

$$X_{ijk} \leq P_j \quad \text{for all } i, j, k, \tag{4}$$

$$X_{ijk} \leq Q_k \quad \text{for all } i, j, k, \tag{5}$$

$$P_{\min} \leq \sum_{j \neq \text{Direct shipment}} P_j \leq P_{\max}, \tag{6}$$

$$Q_{\min} \leq \sum_{k \neq \text{Infeasible site}} Q_k \leq Q_{\max}, \tag{7}$$

$$0 \leq X_{ijk} \leq 1, \tag{8}$$

$$P_j \in \{0, 1\}, \tag{9}$$

$$Q_k \in \{0, 1\}. \tag{10}$$

The objective function minimizes the sum of costs to transfer products from origination sites through collection sites to the destination facilities and the fixed cost of opening the collection and destination sites. All the supply of products available at the origination sites are transported to destination facilities either directly or via collection sites in the network by way of constraint set (1). Constraint set (2) limits the units sent through collection site j to the capacity of site j , and constraint set (3) limits the units ending up at destination site k to the capacity of site k . Constraint set (4) prohibits units from being routed through collection site j unless the site is opened, and constraint set (5) prohibits units from ending up at destination site k unless this site is opened. Constraint (6) ensures that a minimum number of collection sites remain open and the maximum number of collection sites that can be opened, and constraint (7) limits the minimum number of destination sites remain open and the maximum number of destination sites that can be opened. Constraint set (8) requires the decision variable X to be continuous between zero and one, while constraint sets (9) and (10) enforce the binary restriction on the P and Q decision variables.

A weak formulation of this problem can be obtained from aggregating the demand in constraint sets (4) and (5), arriving at the following alternative constraint sets:

$$\sum_i \sum_k X_{ijk} \leq |I|^* P_j \quad \text{for all } j, \tag{4'}$$

$$\sum_i \sum_j X_{ijk} \leq |I|^* Q_k \quad \text{for all } k. \tag{5'}$$

The advantage of this weak formulation, using constraint sets (4') and (5'), is typically a drastic reduction in computational effort associated with finding optimal solutions to the problem.

4. The solution approach

4.1. Observations

Model Refurb is a zero–one mixed integer-linear programming (MIP) problem. A depiction of a solution to this model is given in Fig. 1 below.

In Fig. 1, there are two open destination facilities and three open collection sites. Note that model *Refurb* exhibits a mixed hierarchical structure: that is, fractional demands at origination sites are assigned to collection sites, which are again assigned to the destination sites. In addition, the fractional demand from origination sites can be assigned directly to the destination sites.

Assuming that there are no collection sites, model Refurb reduces to a CPLP. The CPLP is NP-complete (Davis and Ray, 1969), and as such, model Refurb is NP-hard. The use of conventional MIP tools for solving problem Refurb is limited due to (i) the complexity of the problem, and (ii) the large number of variables and constraints, particularly for realistically sized, even fairly small,

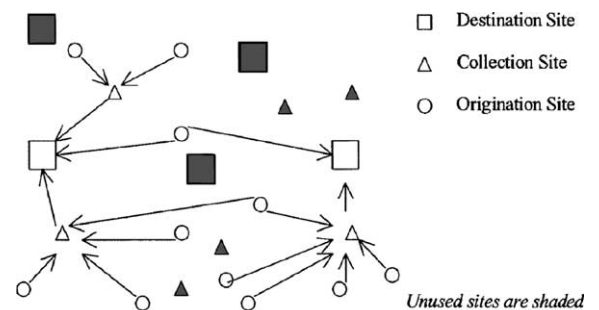


Fig. 1. A depiction of a solution to model Refurb.

problems. This is a difficult problem, and a heuristic solution approach is a viable method for solving this problem.

4.2. The heuristic

Rosing and ReVelle (1997) proposed a heuristic known as “heuristics concentration”, and applied this to the p -median problem. Heuristic concentration (in the context of the p -median problem) is based on running an efficient heuristic numerous times as to identify a subset of the potential facility sites that may warrant further investigation. After repeatedly running a randomized greedy heuristic, the information about which nodes are in the “ p ” set is collected, and a final subset of nodes is used when solving the subset problem to optimality using integer-linear programming. Thus, using a subset of the potential facility nodes results in an optimal solution if all nodes that are in the optimal “ p ” set are in the subset. Also, this subset is limited, and the time to find the optimal solution to this subproblem is only a fraction of that of optimally solving the problem using the full set of potential facility sites. It is our intent to propose a heuristic for solving problem Refurb that takes advantage of the above observations. Rosing et al. (1998) compared heuristic concentration to tabu search and concluded that heuristic concentration is superior to tabu search for the p -median problem.

This paper uses AMPL (Fourer et al., 1995) as a front-end interface to CPLEX, which is the MIP solver for the subproblems. All experiments are performed on a 300 MHz Sun Ultra 30 computer with 128 M of RAM and 1 Gb of swap space, running Solaris 2.6 CDE version 1.2. The procedure *SolveRefurb*, described below, was designed to solve problem Refurb. The algorithm has three (3) components: First is the random selection of potential collection and refurbishing sites, second is the heuristic concentration section, and third is the heuristic expansion (HE) component.

4.2.1. Procedure *SolveRefurb*

Random selection:

Set $MaxIterations = \beta$ (where $\beta = 25, 50, \text{ or } 100$ in our experiments)

1. While $Iterations < MaxIterations$ do:
2. Randomly select a subset of size P_{max} from the collection sites, and Q_{max} from the destination sites (all sites have an equal probability of being selected so as not to prejudice the breadth of the search).
3. Append the AMPL model file in such a manner that only the sites selected in step 2 are considered as potential facility sites.
4. Solve the current problem to optimality using AMPL (with CPLEX).
5. Save the solution and its configuration. If the current solution is better than the best previously found solution, then update best-found solution.
6. End while.

Heuristic concentration:

7. Append the AMPL model file with all collection and destination sites chosen in the best previously found solution. Using the information collected in steps 2–5 from the top 5% best solutions, select the additional $[P_{max} + 2 - \text{the number of sites used in the first best solution}]$ most frequently used collection sites, and $[Q_{max} + 2 - \text{the number of sites used in the first best solution}]$ most frequently used destination sites, and change the AMPL model file to consider these selected facility sites as well.
8. Solve the current problem to optimality using AMPL.
9. If the current solution is better than the best previously found solution, then save the solution and its configuration as the best found solution.
10. Report the best solution found.

Heuristic expansion:

11. Append the AMPL model file with all collection and destination sites reported in the best solution found. Add one collection or destination site not chosen in the best solution found.
12. Solve the current problem to optimality using AMPL (with CPLEX).
13. If the current solution is better than the best previously found solution, then remember the solution and its configuration, but leave the best found solution unchanged for now.

14. Repeat from step 11 until all collection and destination sites have been checked.
15. If improvements are found, use the collection or destination site that yields the largest cost savings and save this solution as the best solution found.
16. Repeat from step 11 until no improvements are found by adding one site at a time.
17. Report the best-found solution.

Steps 1–6 are executed iteratively until we have exceeded a maximum number of iterations (we used 100 in our computational tests). In step 2 we randomly select a set of collection and destination facilities in a uniform manner (all nodes have the same probability of being selected). We then append our AMPL model file, in step 3, to reflect the selected potential facility sites found in step 2 (the new model file simply forces non-selected facility sites not to be used at all). Step 4 executes a call to AMPL, using the original data file, but with a modified model file. In step 5 we record the solution, and also update the best-found solution if necessary. The “best” sites are the sites used in the best-found solution plus the most frequently used sites in the top 5% of all solutions (this number was selected empirically). Using the information collected in steps 1–6, we now select the $P_{\max} + 2$ “best” collection sites, and $Q_{\max} + 2$ “best” destination sites, and change the AMPL model file to consider these selected facility sites only (step 7). The “+2” slack was determined empirically as to create a reasonably large set of sites to be solved efficiently by CPLEX. In step 8 we execute a final call to the AMPL solver, and update the best-known solution in step 9. Step 10 outputs the best-found solution from the complete procedure. Steps 11 through 17 find one-opt greedy heuristic improvements until no such further improvements are available.

4.3. A deterministic heuristic

An alternative deterministic heuristic was designed to replace the *random selection and heuristic concentration* portions of the algorithm above. Designing a deterministic heuristic poses a serious

problem. Consider the following: the problem being solved at each of the random selection iterations is one of (1) selecting facilities to open from the subset of randomly selected available facilities, and (2) assigning routes for the demand. In random selection, we select nodes to be available for use randomly, but then we use CPLEX to select the facilities from this short-list and find an optimal assignment of demand flows (given the open facilities chosen from the available facilities). Considering further that the demand flows may be fractional (all demand from an origination (or retail) site does not need to flow over the same route), there are infinitely many demand routes/combinations for the flow. As such, we do not know of a heuristic that would give us meaningful routing values short of using an optimal solver for the flows. Thus, we developed *procedure CC* (a greedy algorithm) that utilizes CPLEX to define the routing schemes (same as for the random selection with heuristic concentration methods):

4.3.1. Procedure CC

- (1) Rank order all collection sites (P 's) and rank order all refurbishment sites (Q 's) from least to greatest according to the ratio Cost/Capacity.
- (2) Select the $P_{\max} + 4$ and $Q_{\max} + 4$ cheapest Cost/Capacity site alternatives and solve with CPLEX.

HE (as defined above) is then performed using Procedure CC's solution as its starting point. Computational results for these heuristic methods are provided next.

5. Computational results

The following five sets of twenty (20) randomly generated problems were created:²

² The test problems are available from the authors upon request.

Problem set	Number of origination (or retail) sites	Number of available collection sites	Maximum allowed number of collection sites	Number of available Refurb sites	Maximum allowed number of Refurb sites
1	30	14	4	12	2
2	40	20	6	15	4
3	50	30	6	20	4
4	70	30	6	20	4
5	100	40	8	30	6

5.1. Data generation

Each origination, collection, and refurbishing site is randomly located in a 100×100 square. An additional collection site, m , was used as a dummy site to indicate a direct shipment from an origination site to the destination site. An additional destination site with infinite costs and infinite capacity was used to eliminate infeasible solutions. The data generation consists of constructing both costs (fixed and variable) and capacities. The fixed costs are generated by the following formulae: (Note: Square brackets denote random number generation from a uniform distribution in the range indicated inside the brackets)

Collection sites: $F_j := 0.1([0, 10000] + B_j[0, 10])$,

Refurbishing sites: $G_k := 0.1([0, 25000] + D_k[0, 100])$.

The transportation costs are computed as follows:

$C_{ijk} := \alpha(\text{Euclidean distance from } i \text{ to } j \text{ to } k)$

where α is 0.1 when utilizing a collection site (i to j to k), and is 0.4 when shipping direct from the origination site to a refurbishing site (i to m to k , where the Euclidean distance is computed as a direct distance from i to k).

The demand is generated as: $a_i = [0, 500]$, and the capacities are:

Collection sites: $B_j = [0, 6000]$,

Refurbishing sites: $D_k = [0, 30, 000]$.

5.2. Experimental results

The experimental results demonstrate that the HE algorithm produces significant improvement in

solution values. Applying HE to the results of Procedure CC, 100 random selection iterations with heuristic concentration (HC-100), 50 random selection iterations with heuristic concentration (HC-50) and 25 random selection iterations with heuristic concentration (HC-25) for these problem sets clearly illustrate the additional benefit of the HE technique. Both Procedure CC and random selection with heuristic concentration provide starting solutions that the HE technique.

Table 1 provide the details of all solution results for all four heuristic methods. Table 2 provide the computational times for all methods. Table 3 provides a summary comparison of the result averages and a rank ordering. On average, HE generates high quality solutions regardless of the heuristic procedure used to generate starting point solutions. Optimal solutions are obtained for problem sets 1, 2, 3 and 4 by applying CPLEX to model Refurb (using the weak formulation). For problem set 5, only 11 solutions were verified as optimal by applying CPLEX to model Refurb (using the weak formulation); seven (7) problem instances ran out of memory and two (2) hit the 50,000 seconds computational limit. Optimal solutions are obtained in less time than any heuristic procedure for 20, 17, 14 and 5 instances for problem sets 1, 2, 3 and 4 respectively. Average optimal computational times are 6.1, 44.1, 463.3 and 3143.4 seconds problem sets 1, 2, 3 and 4 respectively. Average computational times for the Procedure CC including HE are 20.5, 91.9, 399.1 and 537.2 seconds respectively, demonstrating that computational times deteriorate substantially for the optimal solutions using CPLEX for model Refurb. The computational times for the heuristics

Table 1
Computational results

Problem	Optimality gap											
	100 Ran- dom (%)	HC 100 Random (%)	HE using HC 100 Random (%)	50 Ran- dom (%)	HC 50 Ran- dom (%)	HE using HC 50 Random (%)	25 Ran- dom (%)	HC 25 Random (%)	HC HE using 25 Random (%)	CC Heu- ristic (%)	HE using CC Heu- ristic (%)	
<i>30 Retail; 14 Collection sites (max of 4); 12 Refurb sites (max of 2)</i>												
1	1.64	1.64	0.00	1.64	1.64	0.00	1.64	1.64	0.00	0.00	0.00	
2	2.86	0.49	0.00	2.86	0.49	0.00	7.95	2.11	0.00	0.00	0.00	
3	1.47	1.15	0.00	2.81	1.15	0.00	2.81	1.15	0.00	2.81	1.97	
4	7.04	0.00	0.00	7.04	0.00	0.00	7.04	7.04	0.00	5.48	0.00	
5	30.29	25.25	23.75	42.27	31.21	0.00	42.27	35.57	0.00	0.00	0.00	
6	5.80	1.81	0.86	5.80	1.81	0.86	5.80	2.68	0.86	0.00	0.00	
7	3.48	0.00	0.00	3.92	2.98	0.00	3.92	3.92	0.00	0.00	0.00	
8	5.69	5.69	5.69	13.02	9.37	5.69	13.02	8.54	5.69	0.00	0.00	
9	19.78	2.61	0.00	19.78	6.85	0.00	19.78	19.78	0.00	27.15	0.00	
10	8.28	2.86	0.00	8.61	6.22	5.55	8.61	6.42	5.55	0.19	0.00	
11	10.50	0.00	0.00	10.88	10.66	8.62	10.88	10.88	8.62	6.30	0.00	
12	0.62	0.62	0.00	0.62	0.62	0.00	6.04	6.04	0.00	0.62	0.00	
13	2.03	2.03	0.00	2.03	2.03	0.00	6.16	3.15	0.00	0.00	0.00	
14	14.83	0.00	0.00	25.73	0.00	0.00	25.73	0.00	0.00	0.00	0.00	
15	1.47	0.00	0.00	1.47	1.47	0.00	4.26	3.88	0.00	0.22	0.00	
16	0.00	0.00	0.00	0.00	0.00	0.00	4.40	4.40	0.00	0.32	0.00	
17	3.51	3.27	0.00	3.51	3.51	0.00	3.51	3.51	0.00	0.00	0.00	
18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
19	6.66	5.03	0.00	53.95	26.01	0.44	53.95	26.01	0.44	0.91	0.44	
20	7.10	3.98	0.00	21.98	16.34	0.00	35.82	23.83	0.00	2.30	0.00	
Minimum	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
Maximum	30.29	25.25	23.75	53.95	31.21	8.62	53.95	35.57	8.62	27.15	1.97	
Average	6.65	2.82	1.52	11.40	6.12	1.06	13.18	8.53	1.06	2.32	0.12	
<i>40 Retail; 20 Collection sites (max of 6); 15 Refurb sites (max of 4)</i>												
21	9.74	0.29	0.00	10.55	4.72	0.00	10.55	9.19	0.00	1.15	0.00	
22	2.79	0.00	0.00	2.79	0.59	0.00	2.79	0.59	0.00	0.00	0.00	
23	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.11	0.11	0.11	0.11	
24	11.20	1.02	1.02	17.06	1.78	1.02	7.06	8.16	1.02	0.00	0.00	
25	1.48	1.28	0.20	1.48	1.28	0.20	1.48	0.92	0.17	4.38	0.20	
26	8.94	0.00	0.00	14.69	2.48	0.00	14.69	5.16	0.00	2.81	0.00	
27	5.15	0.30	0.24	5.15	0.30	0.24	5.15	5.15	0.24	0.71	0.00	
28	1.52	0.25	0.00	1.52	0.25	0.00	1.52	0.25	0.00	0.00	0.00	
29	2.50	0.00	0.00	10.81	0.00	0.00	10.81	0.64	0.00	7.05	0.00	
30	5.80	0.00	0.00	5.80	5.80	0.00	5.80	5.80	0.00	0.00	0.00	
31	2.38	2.27	0.00	10.84	1.32	0.00	12.50	8.42	6.77	0.00	0.00	

32	7.26	5.01	0.00	7.26	3.78	0.00	7.26	4.12	0.00	4.07	0.00
33	2.23	0.73	0.73	6.86	5.02	0.73	10.00	9.59	9.59	3.53	0.00
34	0.00	0.00	0.00	4.36	2.55	6.00	4.78	4.78	0.00	3.82	3.04
35	0.00	0.00	0.00	2.02	0.00	0.00	7.00	5.02	0.00	0.00	0.00
36	2.50	0.26	0.00	8.69	3.26	0.00	8.69	2.61	0.00	2.21	0.00
37	0.53	0.00	0.00	0.53	0.00	0.00	0.53	0.00	0.00	0.53	0.00
38	11.00	11.00	6.03	11.00	11.00	6.03	18.14	17.26	6.03	8.72	6.03
39	0.78	0.00	0.00	2.40	1.02	1.02	8.65	3.76	0.00	0.00	0.00
40	4.95	1.40	0.00	4.95	196	0.00	4.99	1.96	0.00	7.78	0.00
Minimum	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00	0.00	0.00	0.00
Maximum	11.20	11.00	6.03	17.06	11.00	6.03	18.14	17.26	9.59	8.72	6.03
Average	4.04	1.19	0.41	6.44	2.36	0.46	7.63	4.68	1.20	2.34	0.47

50 Retail; 30 Collection sites (max of 6); 20 Refurb sites (max of 4)

41	9.71	9.14	0.00	14.28	13.94	0.00	18.84	18.04	0.00	1.39	0.00
42	11.47	6.05	4.03	18.77	13.71	0.00	18.77	16.69	0.00	9.93	0.00
43	3.53	0.00	0.00	4.26	1.82	0.00	16.48	6.31	0.00	8.96	0.00
44	15.03	4.39	0.00	16.58	11.36	0.00	16.58	16.08	0.00	4.05	0.00
45	22.71	5.04	0.00	26.27	9.11	0.00	26.27	8.89	0.00	3.42	0.00
46	3.14	1.34	0.00	7.73	3.76	1.17	7.73	7.03	1.17	1.11	0.00
47	6.13	5.93	0.00	6.13	5.93	0.00	6.13	6.11	0.00	0.05	0.00
48	9.63	9.63	0.44	14.44	9.47	0.44	29.90	9.67	0.00	0.44	0.44
49	4.51	4.51	0.32	12.37	7.55	0.32	12.37	10.45	0.32	6.53	0.32
50	12.59	11.65	0.00	15.47	4.89	0.00	15.47	13.51	8.57	18.03	0.00
51	2.99	2.99	1.86	2.99	2.99	1.86	7.16	7.08	1.86	2.33	1.86
52	32.27	14.69	14.00	33.57	29.07	14.00	33.57	26.42	21.59	10.87	0.87
53	6.93	3.82	0.00	6.93	4.78	0.00	9.83	8.95	0.00	0.00	0.00
54	17.49	6.64	3.77	18.10	13.31	4.60	18.10	18.10	3.77	19.44	3.77
55	30.81	14.37	0.00	42.68	35.37	0.00	42.68	31.02	18.65	1.35	0.00
56	9.68	1.30	0.00	9.68	1.30	0.00	9.68	4.17	0.00	0.00	0.00
57	10.75	6.08	0.29	21.94	9.07	0.29	40.60	16.69	0.29	2.14	0.29
58	5.00	0.00	0.00	5.18	0.00	0.00	5.18	4.46	0.00	0.00	0.00
59	20.45	6.77	0.00	20.45	11.00	0.00	31.00	16.50	0.00	16.58	0.00
60	15.39	10.06	0.00	16.31	8.73	0.00	16.31	11.48	0.00	6.92	0.00
Minimum	2.99	0.00	0.00	2.99	0.00	0.00	5.18	4.17	0.00	0.00	0.00
Maximum	32.27	14.69	14.00	42.68	35.37	14.00	42.68	31.02	21.59	19.44	3.77
Average	12.51	6.22	1.24	15.71	9.86	1.13	19.13	12.88	2.81	5.68	0.38

70 Retail; 30 Collection sites (max of 6); 20 Refurb sites (max of 4)

61	27.18	4.42	0.00	33.52	14.84	0.00	54.75	23.59	4.77	15.19	0.00
62	11.33	5.19	0.50	11.33	5.19	0.50	16.47	14.74	0.50	2.41	0.98
63	9.55	7.75	5.12	9.55	6.80	2.89	9.55	7.90	5.12	11.02	3.16
64	20.38	11.72	3.51	20.38	13.74	3.51	34.25	15.45	0.24	8.37	3.46

Table 1 (continued)

Problem	Optimality gap											
	100 Ran- dom (%)	HC 100 Random (%)	HE using HC 100 Random (%)	50 Ran- dom (%)	HC 50 Random (%)	HE using HC 50 Random (%)	25 Ran- dom (%)	HC 25 Random (%)	HC HE using 25 Random (%)	CC Heu- ristic (%)	HE using CC Heu- ristic (%)	
65	15.29	1.47	0.00	15.29	0.46	0.00	24.32	19.99	2.43	7.73	0.00	
66	20.66	10.19	0.00	23.52	12.90	0.11	23.52	9.10	0.11	2.21	0.11	
67	17.88	1.81	0.00	17.88	9.21	3.90	17.88	11.69	0.00	8.53	3.90	
68	13.66	4.71	0.00	13.66	4.05	1.85	44.29	32.99	2.03	4.55	4.11	
69	42.61	30.82	5.16	42.61	33.45	2.11	42.61	28.90	5.16	20.70	2.11	
70	15.02	2.94	0.02	15.23	8.28	4.75	57.46	44.86	0.02	9.27	0.02	
71	13.47	5.14	0.98	13.47	6.67	0.98	13.47	7.92	0.98	0.11	0.00	
72	14.98	5.52	0.00	18.00	10.85	9.99	23.79	9.54	5.54	5.01	5.01	
73	16.64	12.92	3.39	16.64	16.64	3.39	51.91	43.74	4.92	9.18	4.92	
74	10.79	3.60	0.00	10.79	7.94	0.00	10.79	9.79	0.00	5.97	2.51	
75	38.74	6.98	0.39	38.74	21.77	7.74	45.98	37.31	0.00	5.08	0.39	
76	16.30	3.14	0.00	16.50	15.17	0.00	23.35	19.25	0.00	0.38	0.00	
77	12.13	7.62	0.72	12.13	11.06	0.72	12.13	10.52	0.72	9.62	0.72	
78	1.57	1.45	0.00	4.59	3.22	0.00	8.02	6.02	0.00	1.48	0.00	
79	16.00	15.87	9.40	16.00	15.49	9.40	47.21	27.58	0.01	0.00	0.00	
80	35.24	24.80	0.00	35.24	30.46	0.00	44.39	38.92	20.26	4.42	0.00	
Minimum	1.57	1.45	0.00	4.59	0.46	0.00	8.02	6.02	0.00	0.00	0.00	
Maximum	42.61	30.82	9.40	42.61	33.45	9.99	57.46	44.86	20.26	20.70	5.01	
Average	18.48	8.40	1.46	19.25	12.41	2.59	30.31	20.59	2.64	6.56	1.57	
<i>100 Retail; 40 Collection sites (max of 8); 30 Refurb sites (max of 6)</i>												
81	13.95	13.95	9.39	13.95	13.60	9.39	13.95	13.17	9.39	1.64	1.64	2.43
82	28.08	17.61	0.64	48.47	36.26	0.64	48.47	39.53	0.64	11.94	0.00	0.00
83	18.06	7.06	0.22	18.06	4.28	0.22	18.06	9.60	0.22	6.71	0.00	0.00
84	33.81	21.80	1.26	33.81	23.00	1.26	41.51	40.40	1.26	11.82	1.22	1.22
85	18.84	8.89	0.00	18.84	14.35	0.61	37.57	32.88	12.93	9.03	0.00	0.00
86	18.11	5.89	0.71	18.11	10.55	0.00	18.11	11.97	0.00	3.44	0.00	0.00
87	22.94	18.21	0.00	45.27	23.70	0.00	52.96	28.58	0.00	24.90	0.00	0.00
88	27.41	18.07	11.35	27.41	16.84	4.86	27.41	14.17	11.35	15.57	4.86	5.15
89	25.35	17.30	4.76	25.35	15.09	4.76	25.35	20.18	4.76	18.96	5.36	5.24
90	36.39	15.51	4.13	36.39	30.11	14.81	38.56	12.86	4.18	13.83	6.65	4.75
91	23.78	15.57	0.00	28.30	15.91	4.42	28.30	22.46	4.77	14.60	0.00	0.00
92	24.67	16.12	2.14	24.67	14.76	2.14	24.67	19.62	7.29	17.68	7.29	2.14
93	36.04	10.60	0.00	40.53	34.10	0.00	40.53	36.15	0.00	2.42	0.00	0.00
94	43.40	31.49	4.71	43.40	39.05	4.71	43.40	37.39	7.31	13.61	4.71	8.05
95	16.68	9.07	6.55	21.15	11.68	6.55	36.59	31.13	6.55	10.24	9.12	6.55
96	25.45	15.30	6.43	26.13	21.45	6.43	26.13	19.37	11.90	10.21	6.18	6.34

97	27.06	5.12	0.44	34.63	15.37	0.44	53.53	38.54	0.18	7.84	0.00	0.00	0.00
98	43.93	18.87	2.75	43.93	23.02	4.08	43.93	32.88	4.08	4.59	2.75	2.75	2.75
99	44.64	33.21	26.84	44.64	33.13	28.19	45.03	39.51	26.84	10.00	0.01	0.01	0.00
100	32.02	15.10	4.65	35.15	18.74	9.29	37.39	24.71	6.07	6.37	5.37	4.65	0.00
Minimum	13.95	5.12	0.00	13.95	4.28	0.00	13.95	9.60	0.00	1.64	0.00	0.00	0.00
Maximum	44.64	33.21	26.84	48.47	39.05	28.19	53.53	40.40	26.84	24.90	9.12	6.55	8.05
Average	28.03	15.74	4.35	31.41	20.75	5.14	35.07	26.25	5.99	10.77	2.76	2.18	1.90

presented in this paper are fairly consistent within a given problem size, but increase reasonably with increasing problem size. Because computational times using CPLEX for model Refurb deteriorate substantially as problem sizes become large, efficient heuristic procedures are a necessity for solving these large problems.

As seen in Table 3, on average, the HE procedure substantially improves upon the starting point solutions. HE using Procedure CC is 0.12%, 0.47%, 0.38% and 1.57% from optimality for problem sets 1, 2, 3 and 4 respectively, and is 2.76% from the lower bound for problem set 5. Table 4 illustrates that the optimal solution is found by at least one heuristic method in 20, 18, 15, 12, and 8 instances (100%, 90%, 75%, 60% and 40%) for problem sets 1, 2, 3, 4 and 5 respectively.

As the problem size increases to 50 origination sites, computational time for the CC Heuristic with HE is slightly better on average than using CPLEX for model Refurb with the weak formulation. At the 70 origination site level, the computational time is better on average for all heuristic methods than for CPLEX. At the 100 origination site level, the average and worst case computational time advantage for the heuristic methods compared to CPLEX continues to grow. Solution values are better than or equal to CPLEX for at least one heuristic method for 85% of the problems (17 out of 20) at the 100 origination site level. CPLEX ran out of memory in 35% of the 100 origination site problems (instances 81, 88, 89, 90, 94, 96 and 98 in Tables 1 and 2), and exceeded the 50,000 seconds time limitation in 10% of the cases (instances 84 and 92 in Tables 1 and 2). Thus, we conclude that for very large problems, it is essential to use the heuristic methods.

Consistent with Rosing and ReVelle (1997), heuristic concentration is not deterministically a fast solution methodology. Total processing time using CPLEX for model Refurb with the weak and strong formulation for each problem was limited to 50,000 seconds. Thus processing was cut short for problems where the best solution was not verified to be globally optimal (see Table 1). Problem instance 5 of problem set 1 is truly an outlier, with optimality gap of more than 23% for the HE with the HC-100 random selection method.

Table 2
Computational time

Problem	Computational time																
	100 Random	HC 100 Random	HE using HC 100 Random	Total time	50 Random	HC 50 Random	HE using HC 50 Random	Total time	25 Random	HC 25 Random	HE using HC 25 Random	Total time	CC Heuristic	HE using CC Heuristic	Total time	Weak CPL-EX formulation	Strong CPLEX formulation
<i>30 Retail; 14 Collection sites (max of 4); 12 Refurb sites (max of 2)</i>																	
1	57.7	0.6	24.0	82.3	29.5	0.6	24.6	54.7	14.6	0.6	24.0	39.2	0.8	11.9	12.7	3.1	7.9
2	65.5	1.1	27.3	93.9	32.5	1.0	27.8	61.3	16.6	1.1	42.0	59.7	2.6	12.5	15.1	10.8	2416.9
3	57.1	0.7	24.1	81.9	28.0	0.7	23.5	52.2	14.3	0.7	24.1	39.1	1.1	23.3	24.4	5.7	17.9
4	58.6	0.8	12.1	71.5	29.7	0.8	12.0	42.5	14.7	0.7	24.5	39.9	1.2	24.1	25.3	5.4	513.6
5	57.9	1.1	37.6	96.6	29.1	0.9	37.6	67.6	14.4	0.8	48.8	64.0	1.2	12.0	13.2	4.3	67.8
6	58.3	0.6	23.9	82.8	29.6	0.7	23.9	54.2	14.7	0.7	24.1	39.5	1.4	11.9	13.3	9.0	9.8
7	58.6	0.6	11.8	71.0	28.8	0.6	23.4	52.8	15.2	0.6	24.0	39.8	1.0	11.8	12.8	7.2	20.9
8	56.9	0.8	11.9	69.6	28.6	0.7	24.0	53.3	14.1	0.7	24.0	38.8	1.3	12.4	13.7	8.6	44.4
9	58.5	0.8	25.5	84.8	30.0	0.8	38.6	69.4	15.0	0.7	50.7	66.4	1.7	50.2	51.9	6.6	201.6
10	58.2	1.2	49.2	108.6	29.4	0.9	24.0	54.3	14.7	0.8	23.7	39.2	1.8	24.1	25.9	16.3	795.8
11	57.1	0.7	11.8	69.6	28.5	0.6	23.7	52.8	14.4	0.6	24.1	39.1	1.2	24.2	25.4	3.8	17.2
12	58.4	0.6	24.1	83.1	29.6	0.6	23.7	53.9	14.8	0.6	36.3	51.7	1.3	24.0	25.3	4.4	50.6
13	57.2	0.6	23.1	80.9	28.5	0.7	23.1	52.3	14.2	0.7	35.3	50.2	1.0	11.6	12.6	3.2	55.4
14	58.8	0.8	11.5	71.5	29.3	0.7	11.9	41.9	14.6	0.7	12.3	27.6	1.1	11.8	12.9	5.2	52.4
15	56.2	0.6	11.6	68.4	28.1	0.6	35.0	63.7	14.2	0.6	23.4	38.2	1.1	23.7	24.8	3.3	2.1
16	57.2	0.7	11.7	69.6	28.4	0.7	11.8	40.9	14.2	0.8	24.1	39.1	1.5	23.9	25.4	6.2	48.8
17	58.4	0.7	35.8	94.9	29.1	0.7	35.6	65.4	14.6	0.7	36.1	51.4	1.0	12.0	13.0	2.7	308.6
18	55.9	0.7	12.2	68.8	27.5	0.7	12.1	40.3	13.9	0.7	12.0	26.6	1.4	11.9	13.3	4.8	97.0
19	58.0	0.7	23.5	82.2	28.7	0.7	36.4	65.8	14.6	0.8	36.3	51.7	1.1	24.0	25.1	5.6	110.6
20	58.1	0.7	23.6	82.4	28.5	0.7	47.3	76.5	14.5	0.8	47.8	63.1	1.3	23.2	24.5	5.3	29.1
Minimum	55.9	0.6	11.6	68.4	27.5	0.6	11.8	40.3	13.9	0.6	12.0	26.6	0.8	11.6	12.6	2.7	2.1
Maximum	65.5	1.2	49.2	108.6	32.5	1.0	47.3	76.5	16.6	1.1	50.7	66.4	2.6	50.2	51.9	16.3	2416.9
Average	58.1	0.8	21.8	80.7	29.1	0.7	26.0	55.8	14.6	0.7	29.9	45.2	1.3	19.2	20.5	6.1	243.4
<i>40 Retail; 20 Collection sites (max of 6); 15 Refurb sites (max of 4)</i>																	
21	163.7	2.9	113.4	280.0	81.7	2.0	113.1	196.8	40.0	2.1	226.4	268.5	5.0	153.6	158.6	45.8	208.1
22	149.1	1.6	37.0	187.7	74.9	1.8	112.1	188.8	37.8	1.9	113.4	153.1	2.2	38.0	40.2	8.3	37.5
23	166.9	1.9	37.2	206.0	83.3	1.9	37.2	122.4	41.1	1.8	36.3	79.2	2.4	36.6	39.0	21.0	126.1
24	157.1	2.2	36.0	195.3	80.5	1.9	73.4	155.8	40.8	2.2	110.1	153.1	5.8	40.0	45.8	61.4	741.7
25	169.3	3.4	123.1	295.8	83.6	3.4	122.8	209.8	42.8	4.0	164.4	211.2	12.7	196.4	209.1	177.4	50004.2

26	165.5	1.9	36.0	203.4	79.8	1.9	72.8	154.5	39.8	2.0	108.6	150.4	2.6	109.4	112.0	16.7	585.7
27	166.3	1.9	8.8	177.0	81.7	1.8	72.2	155.7	39.8	1.7	107.9	149.4	3.1	111.1	114.2	31.5	1854.6
28	165.8	1.6	72.3	239.7	79.2	1.6	72.4	153.2	40.9	1.6	72.6	115.1	2.9	36.3	39.2	25.9	617.0
29	167.0	2.3	36.5	205.8	87.8	2.0	36.2	126.0	42.9	1.9	73.0	117.8	3.0	73.0	76.0	15.8	41.1
30	159.7	2.0	36.1	197.8	80.1	1.9	145.7	227.7	39.6	1.7	145.4	186.7	2.9	36.0	38.9	20.3	321.4
31	160.8	2.0	108.3	271.1	82.2	2.1	71.2	155.5	40.3	2.0	73.0	115.3	3.6	35.6	39.2	22.7	161.9
32	186.7	3.2	157.3	347.2	92.9	2.9	117.8	213.6	43.3	2.5	115.8	161.6	5.2	114.4	119.6	62.4	2440.9
33	162.3	2.3	36.6	201.2	81.2	2.0	73.0	156.2	40.8	2.0	36.4	79.2	4.2	149.6	153.8	28.0	805.8
34	187.5	2.0	36.8	226.3	93.3	2.1	74.2	169.6	46.5	1.5	110.8	158.8	4.8	74.7	79.5	34.9	409.7
35	161.1	2.1	36.7	199.9	79.2	2.0	36.6	117.8	41.6	1.8	74.5	117.9	2.9	37.3	40.2	13.3	110.1
36	160.6	2.1	73.3	236.0	77.6	1.9	109.4	188.9	38.4	1.7	109.1	149.2	6.8	108.4	115.2	55.0	8826.9
37	157.8	3.5	36.3	197.6	81.7	2.3	36.1	120.1	40.8	2.3	36.2	79.3	3.9	72.7	76.6	106.7	2633.1
38	166.4	1.9	22.7	191.0	82.4	1.5	183.8	267.7	39.7	2.1	221.8	263.6	3.6	110.8	114.4	30.6	130.2
39	162.5	2.1	36.1	200.7	80.3	1.8	36.8	118.9	39.4	2.0	109.6	151.0	4.2	36.4	40.6	64.7	711.2
40	159.2	2.1	108.9	270.2	80.7	2.0	109.6	192.3	41.3	1.9	108.2	151.4	3.3	183.1	186.4	40.2	220.3
Mini- mum	149.1	1.6	8.8	177.0	74.9	1.5	36.1	117.8	37.8	1.5	36.2	79.2	2.2	35.6	38.9	8.3	37.5
Maxi- mum	187.5	3.5	157.3	347.2	93.3	3.4	183.8	267.7	46.5	4.0	226.4	268.5	12.7	196.4	209.1	177.4	50004.2
Aver- age	164.8	2.3	59.5	226.5	82.2	2.0	85.3	169.6	40.9	2.0	107.7	150.6	4.3	87.7	91.9	44.1	3549.4

50 Retail; 30 Collection sites (max of 6); 20 Refurb sites (max of 4)

41	345.3	4.0	523.3	872.6	172.3	4.0	393.1	569.4	86.4	4.0	786.4	876.8	5.5	394.8	400.3	102.6	6211.9
42	369.3	4.4	268.4	642.1	179.2	4.1	807.8	991.1	90.8	4.0	1068.1	1162.9	6.3	540.2	546.5	535.3	3464.3
43	339.0	4.4	132.2	475.6	169.3	4.7	264.2	438.2	84.5	4.0	530.2	618.7	5.5	664.2	669.7	425.9	2387.2
44	357.5	4.5	670.6	1032.6	173.4	4.0	805.3	982.7	87.2	4.0	938.1	1029.3	5.7	534.7	540.4	252.6	2127.5
45	384.2	9.1	553.9	947.2	193.2	8.1	553.3	754.6	97.4	7.2	414.4	519.0	10.2	554.4	564.6	388.7	24806.0
46	366.7	4.2	403.2	774.1	185.7	4.3	405.0	595.0	96.0	4.7	537.8	638.5	5.3	266.6	271.9	851.1	28334.2
47	344.0	4.4	676.3	1024.7	173.5	4.0	677.4	854.9	84.5	4.0	543.7	632.2	4.9	270.9	275.8	249.4	1204.9
48	377.9	4.2	405.4	787.5	191.6	4.1	543.3	739.0	96.7	4.3	672.7	773.7	5.3	134.7	140.0	139.3	1712.0
49	358.9	3.9	267.2	630.0	179.1	3.6	268.2	450.9	89.3	3.4	271.1	363.8	7.7	266.1	273.8	118.8	2077.1
50	356.8	5.4	685.6	1047.8	178.4	4.0	541.1	723.5	88.9	4.1	546.5	639.5	6.7	545.2	551.9	190.7	6145.3
51	355.5	3.9	270.8	630.2	176.2	3.9	269.7	449.8	87.3	4.5	410.4	502.2	17.3	270.0	287.3	456.1	50009.2
52	346.5	4.1	271.0	621.6	177.7	4.1	408.3	590.1	89.6	3.7	404.2	497.5	5.1	396.0	401.1	412.7	664.5
53	368.3	4.3	527.5	900.1	184.0	4.1	535.1	723.2	92.3	3.5	668.9	764.7	7.9	134.4	142.3	171.1	1524.6
54	355.1	5.4	539.9	900.4	173.2	6.8	820.1	1000.1	85.8	4.7	943.0	1033.5	7.3	846.0	853.3	399.1	2580.2
55	358.2	3.8	660.8	1022.8	180.2	3.9	665.4	849.5	94.0	3.9	556.2	654.1	4.5	271.0	275.5	101.8	272.5
56	349.3	3.5	263.7	616.5	175.6	3.4	263.0	442.0	85.3	3.5	394.3	483.1	5.7	130.6	136.3	103.3	13833.8
57	353.2	5.2	428.3	786.7	179.9	5.4	820.1	1005.4	90.4	5.0	849.7	945.1	11.3	282.5	293.8	3509.8	21217.3
58	371.3	4.1	135.3	510.7	181.8	3.8	135.6	321.2	90.8	4.2	268.8	363.8	5.4	135.3	140.7	203.9	41574.9
59	347.1	4.8	530.8	882.7	171.5	4.1	670.0	845.6	87.7	4.1	807.2	899.0	11.6	673.1	684.7	217.7	557.2
60	379.6	5.1	530.4	915.1	188.7	3.9	659.5	852.1	90.2	3.8	661.4	755.4	5.1	527.0	532.1	435.8	1209.5

Table 2 (continued)

Problem	Computational time																
	100 Random	HC 100 Random	HE using HC 100 Random	Total time	50 Random	HC 50 Random	HE using HC 50 Random	Total time	25 Random	HC 25 Random	HE using HC 25 Random	Total time	CC Heuristic	HE using CC Heuristic	Total time	Weak CPL-EX formulation	Strong CPLEX formulation
Minimum	339.0	3.5	132.2	475.6	169.3	3.4	135.6	321.2	84.5	3.4	268.8	363.8	4.5	130.6	136.3	101.8	272.5
Maximum	384.2	9.1	685.6	1047.8	193.2	8.1	820.1	1005.4	97.4	7.2	1068.1	1162.9	17.3	846.0	853.3	3509.8	50009.2
Average	359.2	4.6	437.2	801.1	179.2	4.4	525.3	708.9	89.8	4.2	613.7	707.6	7.2	391.9	399.1	463.3	10595.7
<i>70 Retail; 30 Collection sites (max of 6); 20 Refurb sites (max of 4)</i>																	
61	508.3	6.0	562.3	1076.6	250.9	5.3	930.0	1186.2	128.2	6.1	941.0	1075.3	10.4	555.0	565.4	483.6	19869.3
62	500.7	8.1	752.3	1261.1	246.0	6.6	744.8	997.4	122.3	7.0	923.4	1052.7	45.7	556.5	602.2	5813.4	50011.7
63	504.6	12.5	739.4	1256.5	255.4	6.4	748.7	1010.5	123.7	5.9	743.3	872.9	21.8	382.6	404.4	3073.6	50016.6
64	495.0	10.4	772.8	1278.2	246.7	6.6	1141.5	1394.8	122.9	6.2	601.1	730.2	20.0	783.6	803.6	2337.6	50018.9
65	492.1	15.2	573.8	1081.1	247.7	7.2	382.0	636.9	125.4	5.9	746.9	878.2	53.1	762.6	815.7	2128.4	50012.3
66	509.7	10.2	1524.8	2044.7	258.2	9.4	1146.5	1414.1	129.3	7.0	1141.7	1278.0	17.9	760.9	778.8	4349.2	50014.4
67	533.3	7.1	573.2	1113.6	266.8	5.9	553.4	826.1	129.5	6.5	950.1	1086.1	8.2	366.7	374.9	728.9	36813.8
68	520.9	11.1	748.0	1280.0	254.6	11.3	589.9	855.8	126.2	6.4	1358.1	1490.7	20.3	373.3	393.6	9829.7	50010.9
69	508.1	13.0	1341.2	1862.3	256.9	14.7	1335.9	1607.5	128.3	9.6	948.4	1086.3	17.1	752.5	769.6	916.8	50021.7
70	495.7	7.2	564.8	1067.7	249.4	9.7	562.6	821.7	123.5	6.1	1315.6	1445.2	16.6	752.1	768.7	9233.5	50014.0
71	503.5	6.6	746.9	1257.0	257.2	6.3	746.2	1009.7	133.8	6.6	933.4	1073.8	9.7	372.4	382.1	1905.8	50016.0
72	499.7	6.8	380.2	886.7	250.1	5.9	376.5	632.5	125.7	6.9	563.6	696.2	15.6	186.4	202.0	681.2	9623.0
73	510.1	13.9	1171.2	1695.2	256.2	7.8	1540.1	1804.1	130.6	8.0	1338.7	1477.3	16.7	389.9	406.6	4120.1	50015.7
74	500.4	5.6	559.6	1065.6	249.0	5.7	933.6	1188.3	125.1	5.4	938.3	1068.8	6.6	565.8	572.4	464.8	19643.6
75	506.0	7.4	956.0	1469.4	254.2	8.9	1174.6	1437.7	128.9	6.0	1346.3	1481.2	15.3	387.4	402.7	7304.6	50011.0
76	505.4	6.3	750.6	1262.3	252.1	6.0	937.6	1195.7	126.9	5.5	1122.1	1254.5	7.1	373.9	381.0	232.5	1481.0
77	489.8	6.8	752.9	1249.5	244.2	7.0	944.9	1196.1	121.8	6.0	940.4	1068.2	12.4	752.4	764.8	2286.4	50016.5
78	515.0	7.9	375.1	898.0	261.1	6.3	566.2	833.6	133.4	5.6	562.7	701.7	6.9	374.2	381.1	202.9	28440.7
79	502.7	7.3	746.9	1256.9	249.0	7.4	567.2	823.6	123.3	6.3	1325.8	1455.4	17.0	185.5	202.5	5956.9	50024.4
80	532.7	6.8	962.9	1502.4	265.7	8.4	1337.8	1611.9	129.8	8.1	952.6	1090.5	15.6	756.6	772.2	637.7	50011.9
Minimum	489.8	5.6	375.1	886.7	244.2	5.3	376.5	632.5	121.8	5.4	562.7	696.2	6.6	185.5	202.0	202.9	1481.0
Maximum	533.3	15.2	1524.8	2044.7	266.8	14.7	1540.1	1804.1	133.8	9.6	1358.1	1490.7	53.1	783.6	815.7	9829.7	50024.4
Average	506.7	8.8	777.7	1293.2	253.6	7.6	863.0	1124.2	126.9	6.6	984.7	1118.2	17.7	519.5	537.2	3134.4	40804.4

100 Retail; 40 Collection sites (max of 8); 30 Refurb sites (max of 6)

81	2433.1	83.6	3986.4	6503.1	1258.9	52.6	3216.6	4528.1	590.7	29.1	4015.7	4635.5	141.3	801.3	942.6	32661.7
82	1891.3	34.1	4728.4	6653.8	1048.7	38.5	5509.1	6596.3	552.6	23.2	7814.5	8390.3	33.7	3968.8	4002.5	5489.0
83	2367.4	34.8	3275.2	5677.4	1044.7	27.6	2390.3	3462.6	536.1	24.7	3256.6	3817.4	87.9	3901.3	3989.2	18353.4
84	1960.3	47.0	5076.6	7083.9	961.1	30.9	5048.6	6040.6	489.9	22.4	5131.0	5643.3	80.0	5238.0	5318.0	50028.6
85	2149.3	29.8	4062.1	6241.2	1074.2	25.4	3956.9	5056.5	531.9	32.4	4731.5	5295.8	112.8	2413.0	2525.8	9575.5
86	2053.1	32.4	2296.7	4382.2	1004.2	33.1	3868.9	4906.2	532.3	23.4	4664.3	5220.0	32.0	2334.0	2366.0	6227.2
87	2279.2	36.7	3869.3	6185.2	1050.0	50.6	3984.9	5085.5	500.8	22.7	4658.1	5181.6	47.4	3933.7	3981.1	2936.3
88	2227.6	45.3	3438.3	5711.2	1094.9	37.5	5388.0	6520.4	556.4	31.5	1775.3	2363.2	79.8	4337.8	4417.6	45235.6
89	2038.3	39.4	7129.3	9207.0	1024.2	31.8	6355.5	7411.5	514.1	30.6	7170.7	7715.4	56.8	5618.7	5675.5	24050.9
90	1829.8	33.0	4052.7	5915.5	990.4	56.8	5012.5	6059.7	506.1	29.6	7492.5	8028.2	91.9	3227.3	3319.2	43297.4
91	2357.2	43.1	5590.9	7991.2	1238.7	27.4	4686.8	5952.9	611.3	21.4	4709.0	5341.7	91.8	5921.3	6013.1	4031.0
92	2170.3	120.1	5967.8	8258.2	1048.8	72.9	6093.1	7214.8	547.4	31.4	5117.8	5696.6	79.0	4204.4	4283.4	50028.8
93	2633.5	65.6	6263.9	8963.0	1178.8	71.7	10017.5	11268.0	738.1	45.2	10632.6	11415.9	174.7	4180.1	4354.8	24715.8
94	1903.6	32.9	4212.1	6148.6	939.3	36.5	4984.9	5960.7	472.8	23.3	6707.4	7203.5	47.4	4265.2	4312.6	24444.6
95	1884.4	66.0	3162.4	5112.8	945.1	24.7	4759.8	5729.6	523.5	30.1	6376.3	6929.9	68.4	3171.5	3239.9	5497.9
96	2600.7	76.0	4121.8	6798.5	1291.3	115.3	5926.8	7333.4	721.1	62.4	6010.4	6793.9	280.7	3368.0	3648.7	39623.1
97	1981.7	24.4	3203.6	5209.7	1011.0	21.2	4742.7	5774.9	528.1	26.2	4967.8	5522.1	48.0	2381.1	2429.1	24768.1
98	2190.4	82.8	5634.3	7907.5	1092.4	34.3	5076.7	6203.4	567.4	39.2	6796.5	7403.1	54.1	2751.0	2805.1	37027.7
99	2292.6	61.2	7306.2	9660.0	1184.1	45.9	6508.4	7738.4	653.7	33.6	5679.1	6366.4	71.3	5460.6	5531.9	34586.7
100	2184.9	36.1	4743.2	6964.2	1136.9	65.6	4685.6	5888.1	530.7	49.3	3919.6	4499.6	55.3	2381.4	2436.7	13929.8
Mini- mum	1829.8	24.4	2296.7	4382.2	939.3	21.2	2390.3	3462.6	472.8	21.4	1775.3	2363.2	32.0	801.3	942.6	2936.3
Maxi- mum	2633.5	120.1	7306.2	9660.0	1291.3	115.3	10017.5	11268.0	738.1	62.4	10632.6	11415.9	280.7	5921.3	6013.1	50028.8
Aver- age	2171.4	51.2	4606.1	6828.7	1080.9	45.0	5110.7	6236.6	560.3	31.6	5581.3	6173.2	86.7	3692.9	3779.6	24825.5

Table 3
Summary of average optimality gaps

Heuristic method used to generate starting solution for heuristic expansion technique	Heuristic expansion percentage optimality gap			
	Solution from HC		Final solution	
	Rank	Average (%)	Rank	Average (%)
<i>30 Retail; 14 Collection; 12 Refurb sites</i>				
CC Heuristic ^a	1	2.32	1	0.12
HC 100 Random	2	2.82	2	1.52
HC 50 Random	3	6.12	3.5	1.06
HC 25 Random	4	8.53	3.5	1.06
<i>40 Retail; 20 Collection; 15 Refurb sites</i>				
CC Heuristic ^a	2	2.34	3	0.47
HC 100 Random	1	1.19	1	0.41
HC 50 Random	3	2.36	2	0.46
HC 25 Random	4	4.68	4	1.20
<i>50 Retail; 30 Collection; 20 Refurb sites</i>				
CC Heuristic ^a	1	5.68	1	0.38
HC 100 Random	2	6.22	3	1.24
HC 50 Random	3	9.86	2	1.13
HC 25 Random	4	12.88	4	2.87
<i>70 Retail; 30 Collection; 20 Refurb sites</i>				
CC Heuristic ^a	1	6.56	2	1.57
HC 100 Random	2	8.40	1	1.46
HC 50 Random	3	12.41	3	2.59
HC 25 Random	4	20.99	4	2.64
<i>100 Retail; 40 Collection; 30 Refurb sites</i>				
CC Heuristic ^a	1	10.77	1	2.76
HC 100 Random	2	15.74	2	4.35
HC 50 Random	3	20.75	3	5.14
HC 25 Random	4	26.25	4	5.99

^aThe CC Heuristic is a stand-alone greedy algorithm and does not utilize heuristic concentration.

Table 4
Analysis of optimal solution performance

	Number of optimal solutions found				Found with at least one heuristic
	HE using HC 100 random	HE using HC 50 random	HE using HC 25 random	HE using CC heuristic	
Problem set 1	17	15	15	18	20
Problem set 2	15	14	13	16	18
Problem set 3	13	13	12	14	15
Problem set 4	10	6	5	7	12
Problem set 5	4	3	3	8	8

Note: Optimal solutions are known for all (20) instances in problem sets 1, 2, 3 and 4. Optimal solutions are verified for 11 out of 20 instances in problem set 5.

Our hypothesis is that this starting point solution for HE was stuck in a very poor local minimum.

Even though the HE technique cannot always substantially improve every solution, Table 4 il-

illustrates that optimal solutions were found in a significant number of cases as summarized in Table 4.

As shown in Table 3, the percentage improvement resulting from HE is, on average, substantial. Also, we see that the average improvements are substantial for all of the starting point heuristics. Thus, we conclude that the HE procedure can be an extremely beneficial add-on technique when solving complex location-allocation problems such as model Refurb. A good starting point solution is very important to obtaining good final solutions with the HE technique. From Table 3 we observe that a larger sampling of starting solutions is beneficial for heuristic concentration, but also seems to have a general positive impact on the final solutions after HE. This paper also presents a starting point heuristic for model Refurb, named Procedure CC, that performs very well on most of the problem sets as shown in Table 3. Procedure CC is, with the exception of the 40 retail site problems, the best stand-alone heuristic procedure investigated in this research. HE yields overall superior results when using Procedure CC to obtain the starting point solutions, especially as the problems become larger. Recall from above that Procedure CC is a greedy algorithm that utilizes CPLEX to define the routing schemes (same as for the random selection with heuristic concentration methods). Procedure CC utilizes knowledge of the costs and capacity structure of the problem, rather than relying on an analysis of repetitive random selections as is done in the random selection with heuristic concentration methods. It seems very reasonable that the proper use of problem specific knowledge would yield a very fast heuristic that would provide starting point solutions competitive with (and often better than) methods based on random selection.

6. Summary and conclusions

In this paper we proposed models and solution procedures for a reverse distribution problem. The complexity of the proposed model was such that a heuristic solution procedure was the only viable approach to solve very large problems. We pro-

posed heuristic concentration procedures combined with HE to solve this problem. Optimal solutions were found for a significant proportion of problems. Very large problems can indeed be solved in a reasonable amount of time with the heuristics, whereas they cannot be solved with conventional MIP tools within a reasonable amount of computational time. The proposed solution approach can easily be adapted to other hierarchical facility location and distribution problems, such as the forward logistical flow faced by a retail chain doing inbound consolidation where products flow from many vendors to inbound consolidation centers, and from there to plants or distribution centers.

This paper contributes to the reverse distribution literature first by developing a strong and a weak formulation for reverse distribution logistical problems that includes product recall, product recycling and reuse, product disposal, and hazardous product return. Secondly, this paper adapts the heuristic concentration procedure to solve this very complex problem and provides a new solution methodology (HE). Finally, we studied the impact of the initial node selection criteria on the combined heuristics, and found that our proposed HE improved overall greatly on any starting point solution, but exhibited a superior performance when given higher-quality starting solutions.

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